

The Fatigue Damage Spectrum

Part I: Fatigue Damage

Part II: FDS Generation

Part III: Working with FDS



Part I: Fatigue Damage

- Not fatigue damage: Dropping a glass vase and the vase shattering.
 - This involves exceeding the vase's instantaneous stress limits.
- Fatigue damage: Bending a paperclip over and over until it breaks.
 - The accumulation of damage to a product over time due to the application of repeated loads, repeated stress-inducing vibration patterns which weaken the product.

J. A. Ewing and J. C. W. Housley

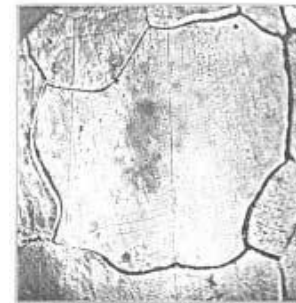


Fig. 9. Specimen after 1000 reversals of a stress of $12.4 \text{ tons per sq. inch}$. $\times 1000$.

Phil. Trans., A, vol. 200, Plate 9.



Fig. 10. Specimen after 5000 reversals. $\times 1000$.



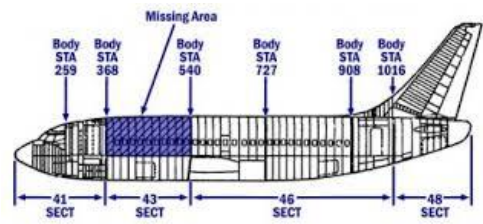
Fig. 11. Specimen after 10,000 reversals. $\times 1000$.



Fig. 12. Specimen after 40,000 reversals. $\times 1000$.

Examples

Aloha Airlines Flight 243
April 28, 1988 – Boeing 737
Fatigue cracking leading to
explosive decompression



Versailles Rail Accident
May 8, 1842 – Locomotive
Axle snapping due to fatigue

The Paperclip

- Suppose we bend a paperclip at three different stress levels.
 - Stress level 1: Bending the outside wire 30 degrees from rest and bending it back.
 - Stress level 2: Bending and bending back 60 degrees.
 - Stress level 3: Bending and bending back 90 degrees.
- After experimenting with each stress level, we produce this table:

Stress Level	Number of Cycles Needed for Paperclip Breakage (Failure)*
1: 90 degree bend and back	16.5
2: 180 degree bend and back	7
3: 270 degree bend and back	4.5



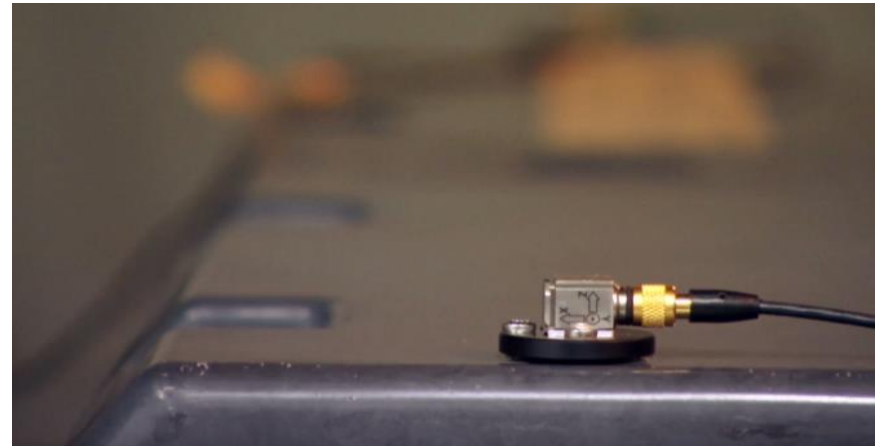
*Non-whole numbers arose from averaging.

*Small Example, **Big Application***

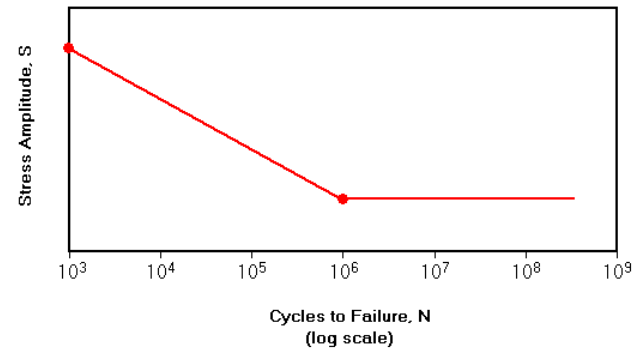
- “The major cause of items failing to perform their intended function is material fatigue and wear accumulated over a time period as a result of vibration-induced stress” (MIL-STD-810G 514.6A-3).
- “All of the tests...consume vibratory fatigue life” (MIL-STD-810G 514.6A-1).
 - Sine, random, shock, etc.

3 Key Questions

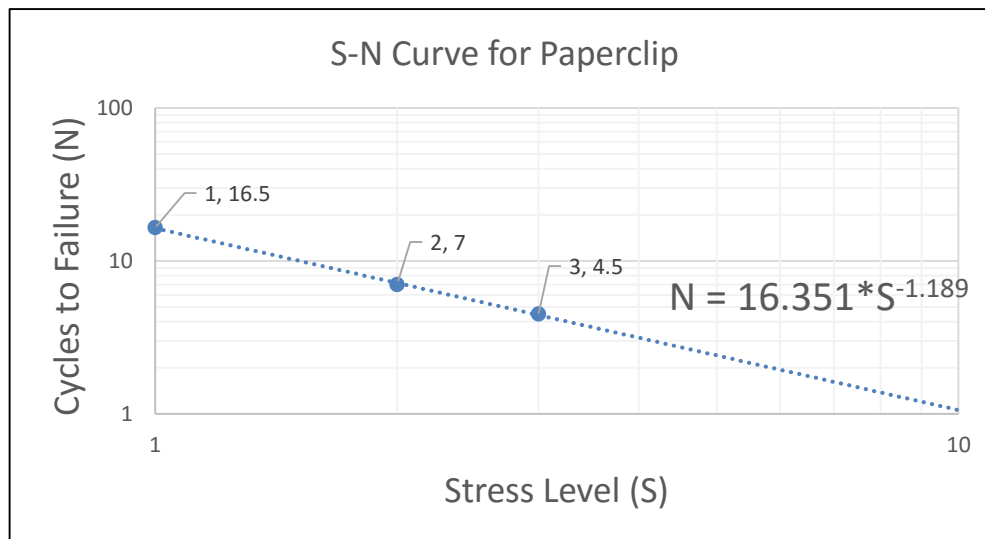
- *Is it possible to design a test (profile and length of time) that accurately simulates the end use environment of a product through its lifetime?*
- *Is it possible to confidently combine multiple, complex end use environments into a single test profile?*
- *Is it possible to reliably accelerate a test and maintain an accurate simulation of imported field data?*



The S-N Curve



- A plot of stress (S) versus the number of cycles (N) required to cause product failure at that stress level—quantifies fatigue damage.
 - The S-N curve is usually plotted with logarithmic scaling, due to the nature of fatigue damage.
 - Idealized shape—a line on the log-log plot → Idealized equation: $N = cS^{-b}$



Stress Level 1: 90 degrees
Stress Level 2: 180 degrees
Stress Level 3: 270 degrees

Palmgren-Miner Rule

“Most often, the linear Palmgren-Miner (P-M) rule is employed to compute a fraction of total life ‘consumed’ by the load. The rule postulates that the order of cycles is irrelevant and that the total damage is a sum of the damages due to individual cycles. One predicts the fatigue failure if the accumulated damage exceeds some critical threshold” (Rychlik 9/14).

In symbolic form,

$$D = \sum_i \frac{n_i}{N_i}$$

where:

n_i = number of cycles applied with peak stress S_i

N_i = number of cycles with peak stress S_i needed to cause failure

D = total damage (failure occurs when $D \approx 1$) (Henderson and Piersol 21).

Calculating Fatigue Damage

1. $D = \sum_i \frac{n_i}{N_i}$ (Palmgren-Miner rule)
2. $N_i = cS_i^{-b}$ (Equation of idealized S-N curve)
3. $D = \sum_i \frac{n_i}{cS_i^{-b}}$ (Substitution)

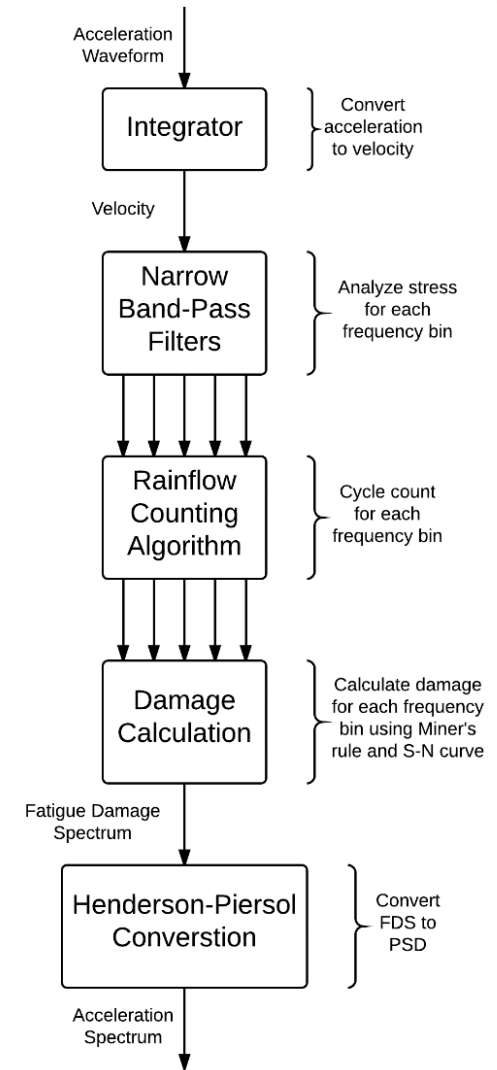
$$D = \frac{1}{c} \sum_i n_i S_i^b$$

Damage can be calculated with knowledge of...

- The S-N curve (the slope, b , primarily)
- The cycles and amplitudes present in a waveform

Part II: FDS Generation

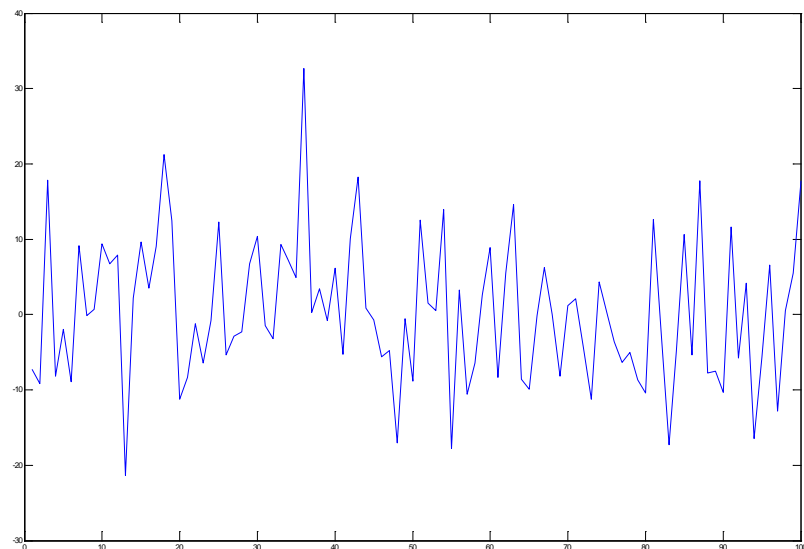
- Acceleration waveform is converted to velocity.
 - “Velocity...has a direct relationship to stress” (Henderson and Piersol 21).
- Waveform is passed through narrow-band filters.
 - Cycle counting and damage calculation ensue in each frequency-bin to determine damage contributions over a spectrum of frequencies.
 - Q value determines filter width
 - Spectrum spacing determines number of points in spectrum (number of frequency bins)



Rainflow Cycle Counting

- Damage analysis requires knowledge of the cycles and amplitudes present in a waveform.
- Counting cycles of a sine wave isn't difficult. But a random wave? Something else?
- Rainflow Cycle Counting
 - Breaks down a waveform—no matter how “messy”—into its constituent cycles and amplitudes.

Cycles



Cycles

Minima	Maxima	Amplitude (Magnitude)
-8.2040353	-1.967113379	3.118460957
9.10748614	-0.122652498	4.615069317
6.75237338	7.859685241	0.553655928
-8.9014374	9.394163082	9.147800251
9.66285539	3.499941084	3.081457152
-1.2177874	-6.425910565	2.604061585
9.33594674	4.881848134	2.227049302
10.3684	-3.170813439	6.76960672
12.2811277	-5.313658352	8.797393024
21.2118842	-11.23185797	16.22187108
0.31700758	3.412749345	1.547870884
-0.8353099	6.16421091	3.499760399
-5.2499919	18.29102237	11.77050715
-5.6346541	-4.735740583	0.449456743
-0.5372005	-8.81344066	4.138120088
12.5589143	0.549326807	6.004793765
-17.035345	13.98552443	15.51043481
3.29654062	-10.57902771	6.937784165
8.91694304	-8.366224107	8.641583574
-8.1632674	2.070686649	5.116977012
-9.9212262	6.269107348	8.095166757
-6.3123748	-5.003249846	0.654562457
4.35357963	-10.40093157	7.377255596
-11.204897	12.65431472	11.92960607
10.6038432	-5.381539717	7.99269148
14.5840932	-17.29031417	15.93720368
-7.7951879	-7.529719231	0.132734344
-5.8010606	4.173023879	4.987042256
-10.331334	11.63838518	10.98485946
6.59232042	-12.80401598	9.698168197
17.7733767	-16.48125021	17.12731348
-9.1632727	17.87553175	13.51940223
-21.327407	32.66198728	26.9946971
-17.756484	17.78446713	17.77047545

Damage Calculation

$$D = \frac{1}{c} \sum_i n_i S_i^b$$

- c is given the value 1 (more on this later)
- Having an accurate material parameter (b), the cycles, and each cycle's corresponding magnitude, damage is accumulated.
 - Nuance: Instead of calculating damage in lumps of n_i , damage can be counted one cycle at a time ($n_i = 1$ for each cycle):

$$D = \frac{1}{c} \sum_i S_1^b + S_2^b + S_3^b + S_4^b + \cdots S_n^b$$

- The resultant damage values from each frequency bin are plotted on a damage vs. frequency plot—the fatigue damage spectrum (FDS).

Henderson-Piersol Conversion

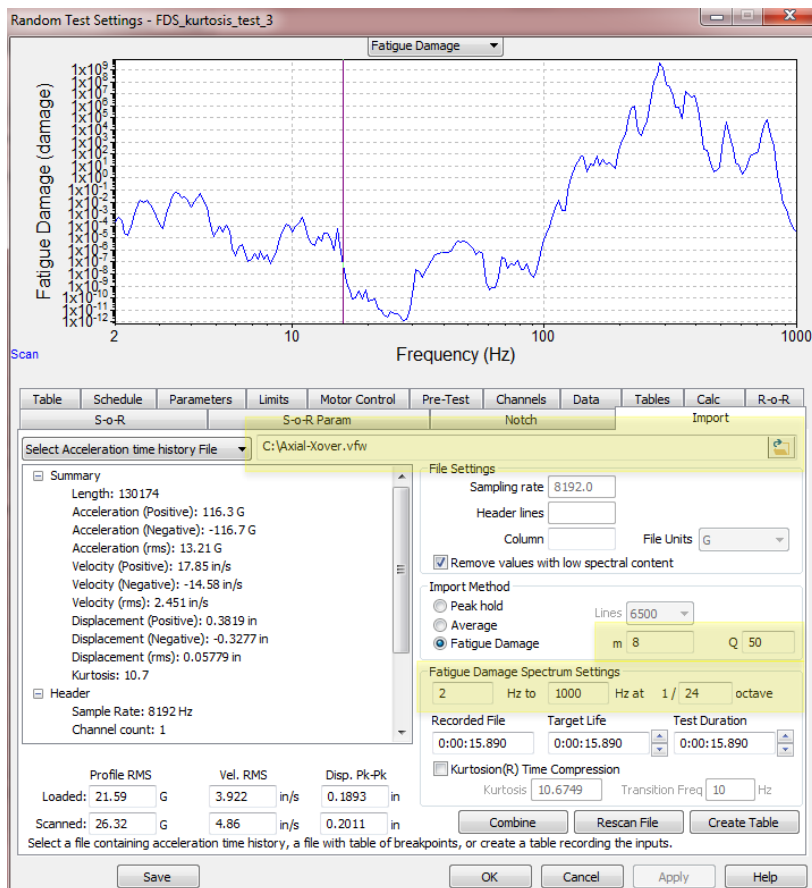
- Using the work of Henderson and Piersol, the FDS is converted into its corresponding power spectral density (PSD).
 - This PSD applies the same amount of damage to the product as did the original, imported waveform(s).
 - Nuance: Output of conversion is a Gaussian (kurtosis = 3) waveform. Kurtosion® Time Compression can be utilized post-conversion to adjust kurtosis of test.

Part III: Working with FDS

How can I use FDS?

1. Generate a random test profile from multiple environments Example: G.M. challenge
2. Compare product vibration in the field to test currently run on a shaker Example: Ford Mustang fuel rail test.
3. Compare a new environment to an existing environment. Does not require a shaker. Example: engine computer comparison in 2 vehicles
4. Compare damage between 2 tests. Example: 2 random tests or RS and ED shakers
5. Envelope different vibration specs from different customers. Example: 5 customers buy my alternator, and they all have different vibration specs. Let us agree on one spec.

Part III: Working with FDS



- How to Generate a Random Profile to Simulate End-Use Environment
 1. Enter the product's material parameter.
 2. Set filter width (Q value) and spectrum spacing for narrow-band filtering.
 3. Set frequency range of import.
 4. Import waveform that accurately represents the product's end-use vibration environment.

Setting Target Life

Fatigue Damage Spectrum Settings

2 Hz to 1000 Hz at 1 / 24 octave

Recorded File Target Life Test Duration

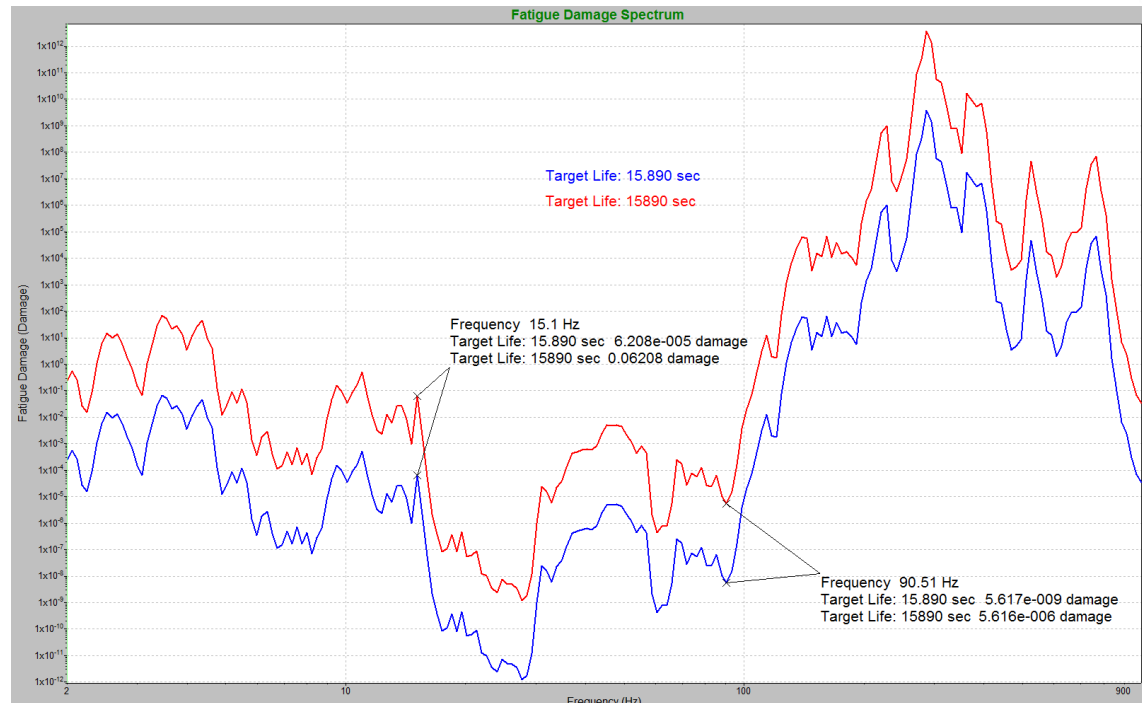
0:00:15.890 0:00:15.890 0:00:15.890

☐ Kurtosis(R) Time Compression

Kurtosis 10.6749 Transition Freq 10 Hz

5. Set target life

- Linearly increases values in FDS by same factor with which waveform time-length was increased.
- When target life is increased by a factor of k beyond the length of the imported waveform, the software “reckons” that the product would be subject to that original waveform k times throughout its life—that waveform repeated k times. Thus, the damage present in the original waveform is multiplied by a factor of k .



Setting Test Duration

Fatigue Damage Spectrum Settings

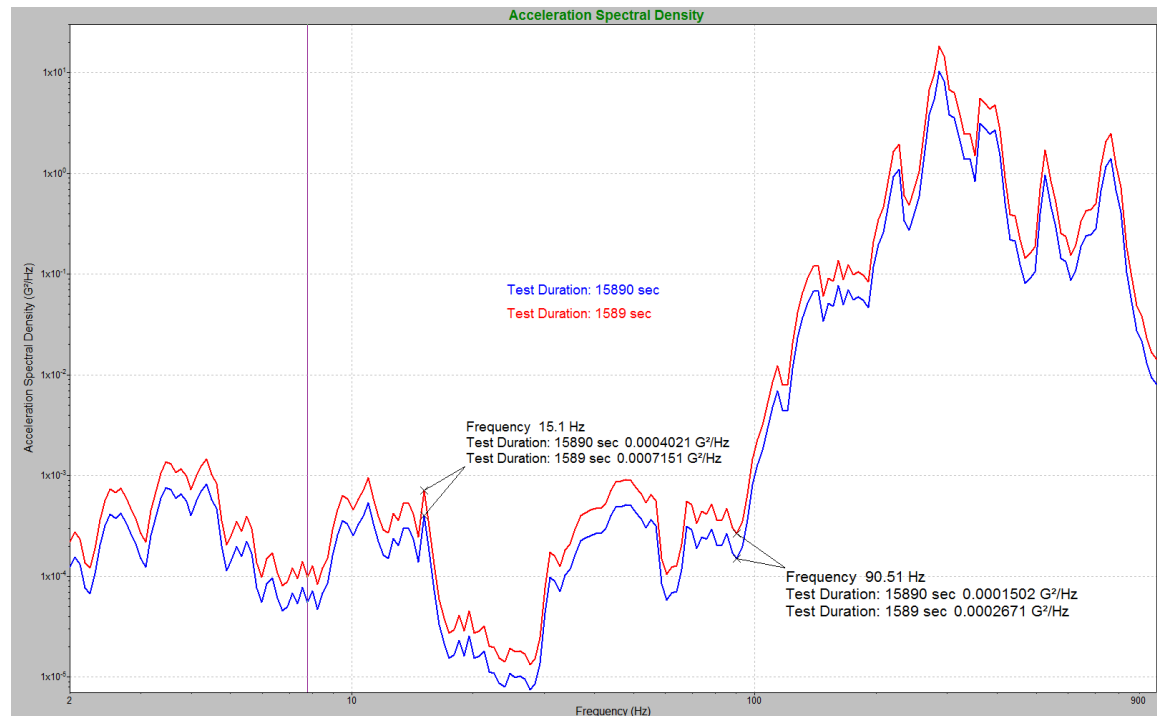
2 Hz to 1000 Hz at 1 / 24 octave

Recorded File Target Life Test Duration

0:00:15.890 4:24:50 0:26:29

☐ Kurtosis(R) Time Compression

Kurtosis 10.6749 Transition Freq 10 Hz



6. Set test duration

- Reducing test duration (while keeping target life the same) applies the same amount of fatigue damage, but more quickly.
- Naturally, the same damage application in less time means a higher acceleration PSD—this according to the time-power relationship (previous slide).
- With time reduction, test acceleration increases non-linearly (notice the power $\frac{m}{2}$ in the previous slide), and depends on the material parameter.

Final Steps

7. Kurtosion® Time Compression

- Although the converted PSD outputs a Gaussian (kurtosis = 3) waveform, Kurtosion® Time Compression can be used to adjust the kurtosis of the test.
- Since Kurtosion® incorporates more high-amplitude peaks (damage increases with amplitude) into the test than does a Gaussian test, while still maintaining the same test level, with Kurtosion® Time Compression the same amount of damage can be applied with a test at a lower level. We adjust this factor automatically.
- This step involves setting the kurtosion value and the transition frequency.
- The main reason to add Kurtosion is to more closely replicate the actual environment.

Assumptions of damage will be more accurate if you are closer to the real life distribution.

Fatigue Damage Spectrum Settings

2 Hz to 1000 Hz at 1 / 24 octave

Recorded File Target Life Test Duration

0:00:15.890 4:24:50 0:26:29

☒ Kurtosion(R) Time Compression

Kurtosis 10.6749 Transition Freq 10 Hz

Combine Rescan File Create Table

Combining Waveforms

- Q. What if a product's environment (or a test designed to test a product) is composed of multiple waveforms—multiple vibration patterns? How do damage and FDS apply?
- A. Each waveform, each vibration pattern—in fact, every cycle of every vibration—adds damage. FDS allows one to combine the damages of multiple waveforms and generates the corresponding PSD which applies the same amount of damage present in the original waveforms.

Further, each waveform's target life (its contribution to total life) can be individually set.

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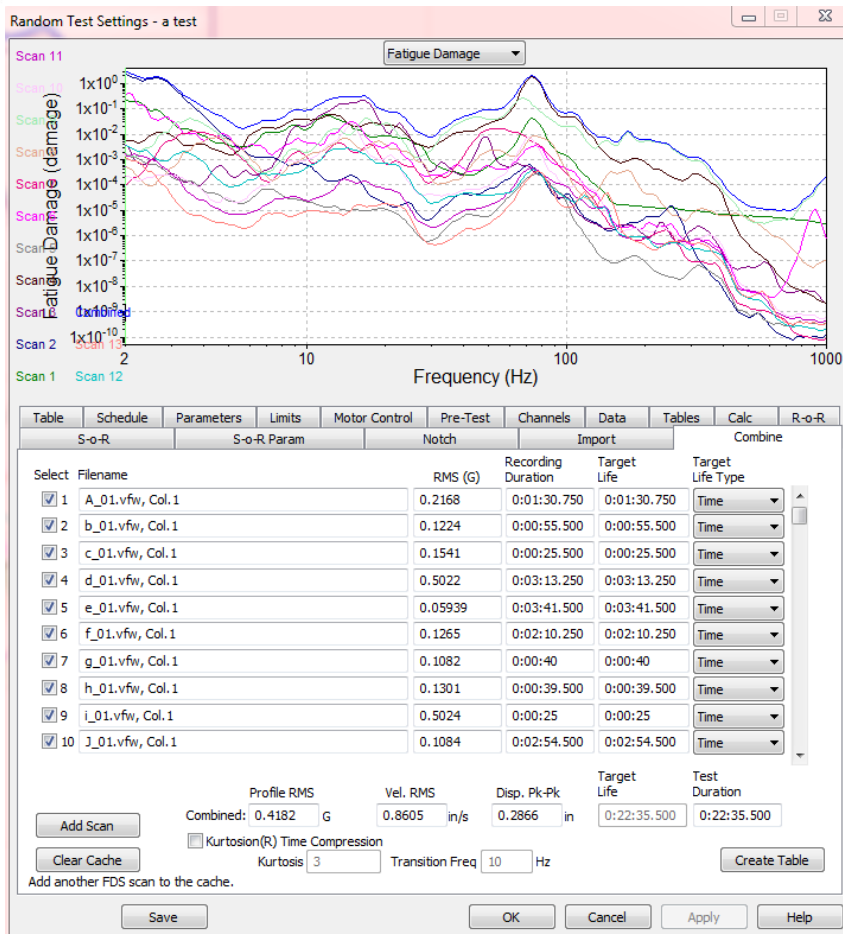
The GM Challenge

- GM assembled a composition of waveforms, each with its own target life, that represented the vibrations the product would experience in its expected/desired lifetime.
- GM provided this composition as well as the quality factor (Q) and material parameter (b) of the product.

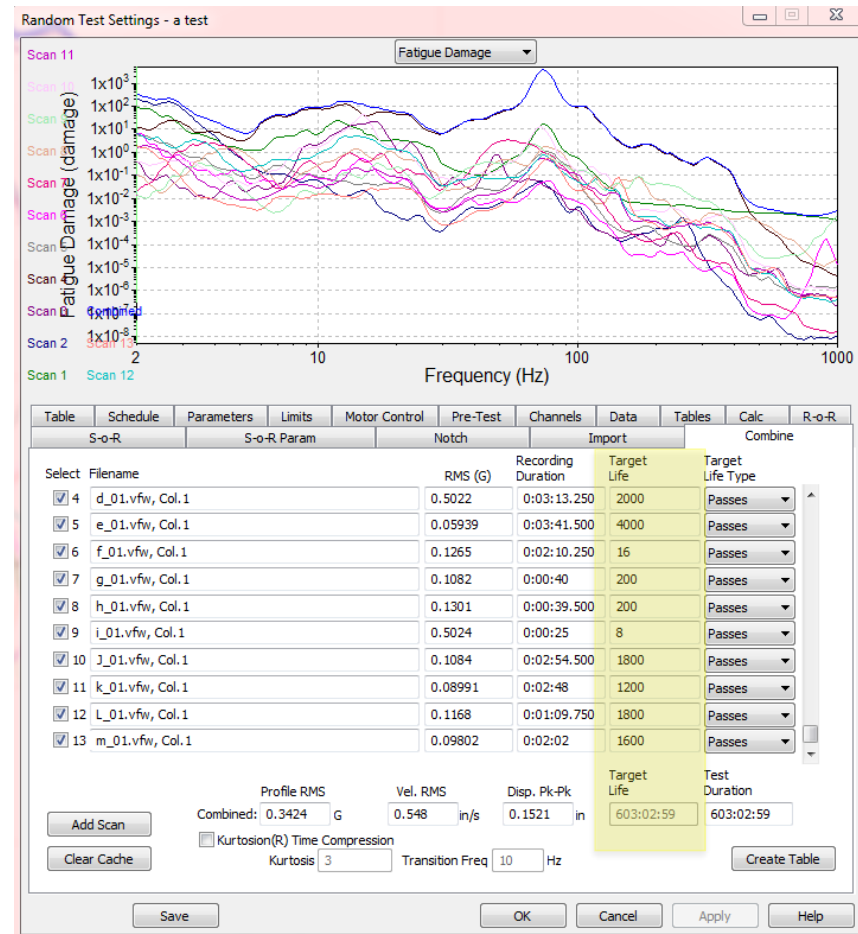
INPUTS		
FILENAME	REPETITIONS	Dyanmic amplification, $Q = 10$ Spring stiffness, $K = 1$ SN coefficient, $A = 1$ SN coefficient, $C = 1$ SN exponent, $b = 4$
A	400	
B	90	
C	100	
D	2000	
E	4000	
F	16	
G	200	
H	200	
I	8	
J	1800	
K	1200	
L	1800	
M	1600	

The Challenge: Combine these in FDS and see how they compare with the results of GM's current method.

The GM Challenge

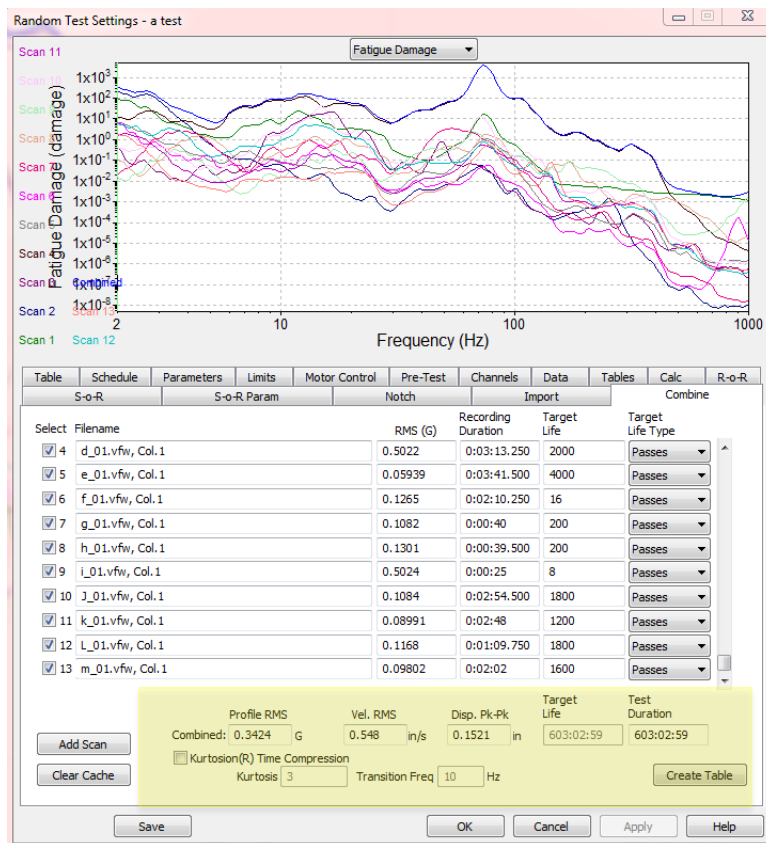


Importing 13 Waveforms into FDS

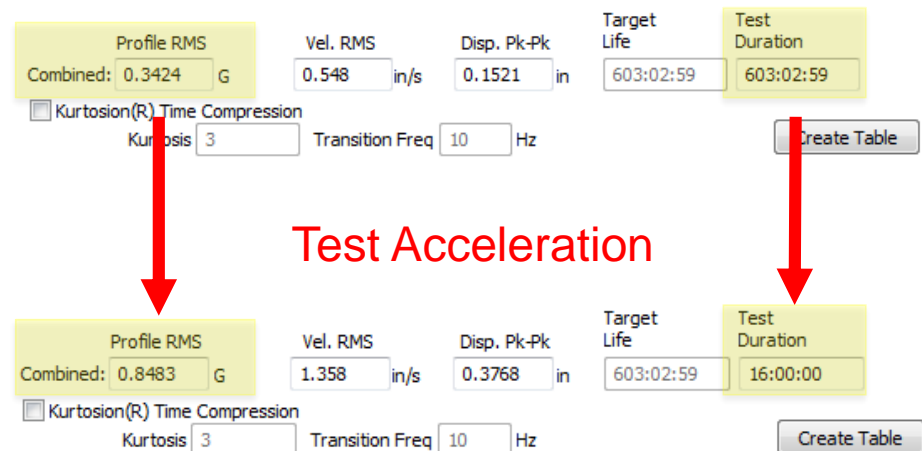


Extending Target Lives

The GM Challenge



- 603 hours equals 25 days. That's a lot of time, and time is money.
- Unsurprisingly, GM desired a shorter test—16 hours.



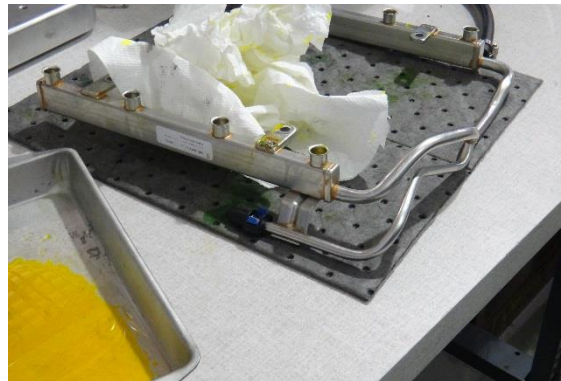
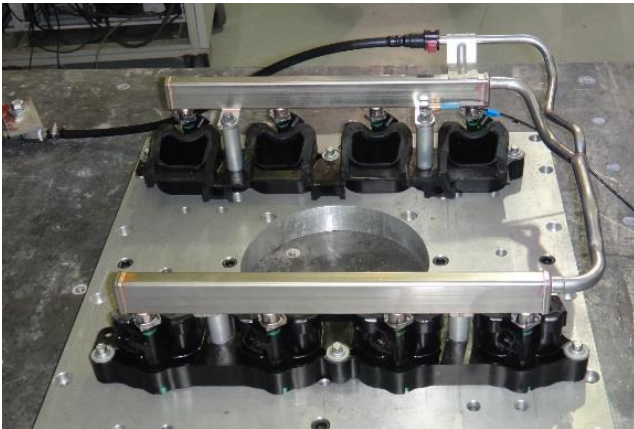
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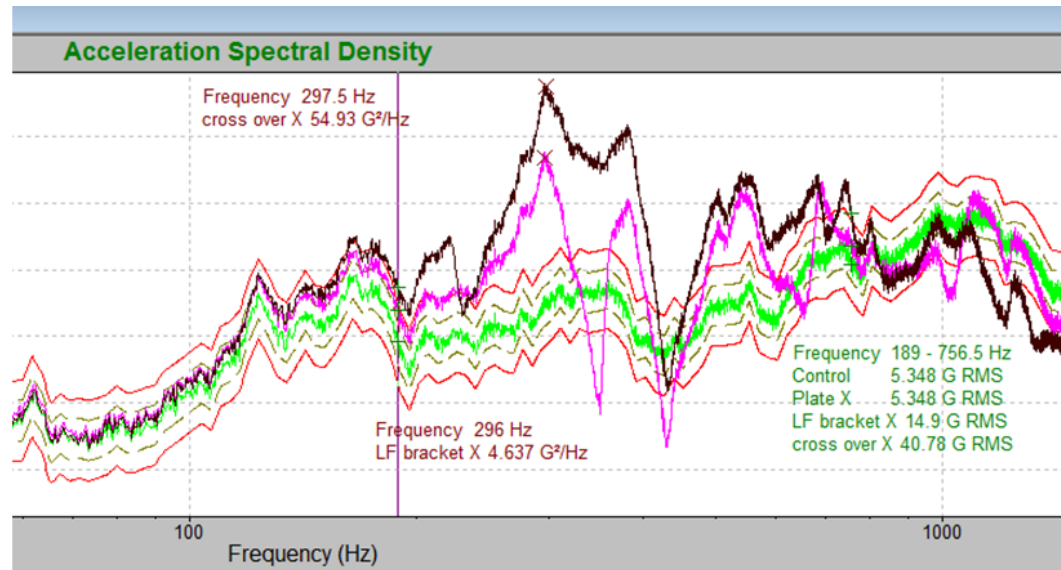
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The Ford Case Study

- The Problem: Lab technicians at a Ford Motor Company testing facility faced a problem. A fuel-rail on their 5.0 L and 6.2 L BOSS engines experienced several failures on the dynamometer.
 - These failures had never been observed in the field.
- Would these failures occur in the field, or were their laboratory tests simply over-testing the fuel rail?



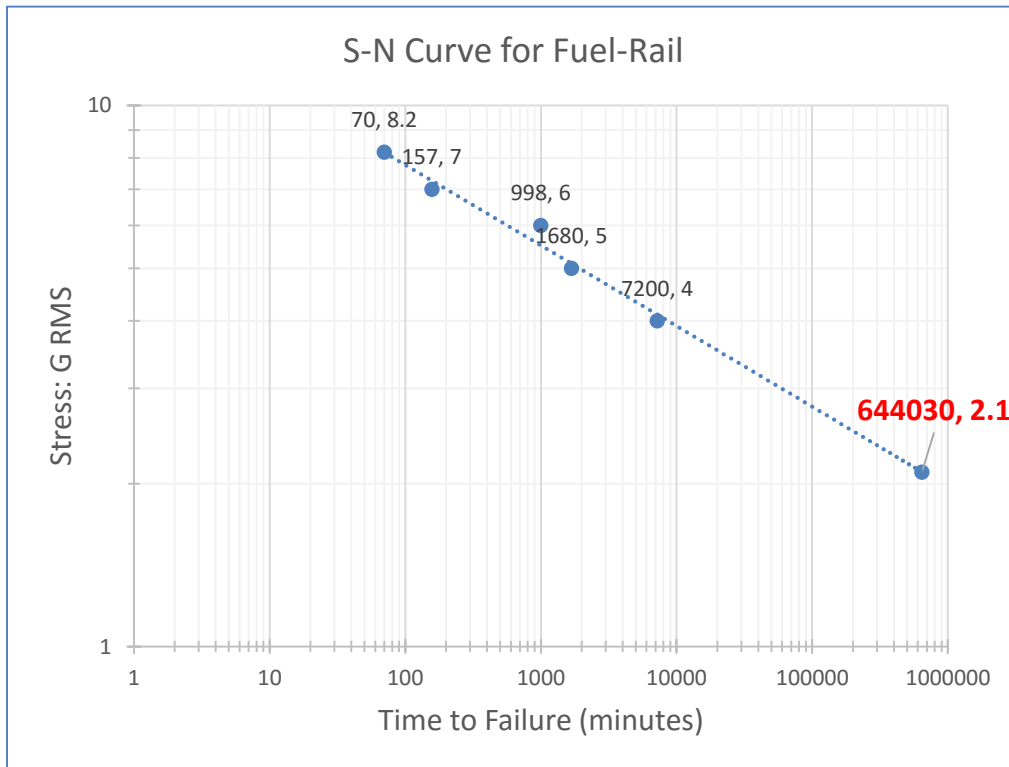
Analyzing Data



- Gathered data from various parts of the engine (the environment applying vibrations to the fuel-rail).
- Concentrated specifically on the engine head.
 - This area provided the largest G RMS values in response to the test.
 - This area appeared to be putting the energy into the “crossover” component of the fuel-rail system, which component contained the most severe vibrations.
- Concentrated on 243-423 Hz band—region of strongest resonance (and resonances are primary sources of damage).

Fuel-Rail S-N Curve

- Used data from tests on Ford's fuel-rail system to develop an S-N curve.



After extrapolating the S-N curve to the 2.1 G RMS range (extrapolated point in red), the extrapolated curve indicated that if the fuel-rail system vibrated at 2.1 G RMS for the 243-423 Hz range some 10,000 hours of testing would be required before failure—i.e., overtesting.

Solution: Finding the Right Test

- Sine test at resonances?
 - Concerns: Which resonances, at what amplitude? How is this correlated to “life”?
What about shifting resonances?
- Random test?
 - Which test profile, at what amplitude? How is this correlated to “life”?
- Fatigue Damage Spectrum
 - Tests a product by applying the amount of damage it would experience in its desired/expected lifetime.
 - Applies damage via a corresponding random test.

Developing a Test with FDS

- Already had a waveform containing the engine head vibration patterns (which caused significant resonances in the fuel-rail).
- Already developed an S-N curve, which afforded an approximation of the material parameter.
- Order of operations:
 - Import the bandlimited waveform with proper m .
 - Set the target life to Ford's expected/desired lifetime for the fuel-rail.
 - Set the desired test duration.
- This would apply the fuel-rail's life-dose of damage.
- This could be used in tandem with Kurtosion®.

Case-Study Qualifications

- There are several engineering common sense questions, the answers of which could skew this test conclusion.
 - Is the nasty engine run-up waveform a good data point for the life test?
 - Is the 120 hour life and 240 hour over test a good bench-mark?
 - Is band limiting the test, and only focusing on the known resonances in the range of 243-423 Hz too limiting? Is there hidden potential damage outside this narrow range that is undiscovered? We may need a bigger shaker to test a wider band of damage potential.
- For FDS in general, there is a limit to how far a test can be accelerated.
- And, of course, the accuracy of the FDS method depends on the accuracy of the material parameter m .

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Step-by-Step

The following section is an in-depth description of the way we arrived at our conclusions.

1. We selected and set up the vehicles.

We chose two Mustangs to use for our comparisons – red 2014 and green 2001.

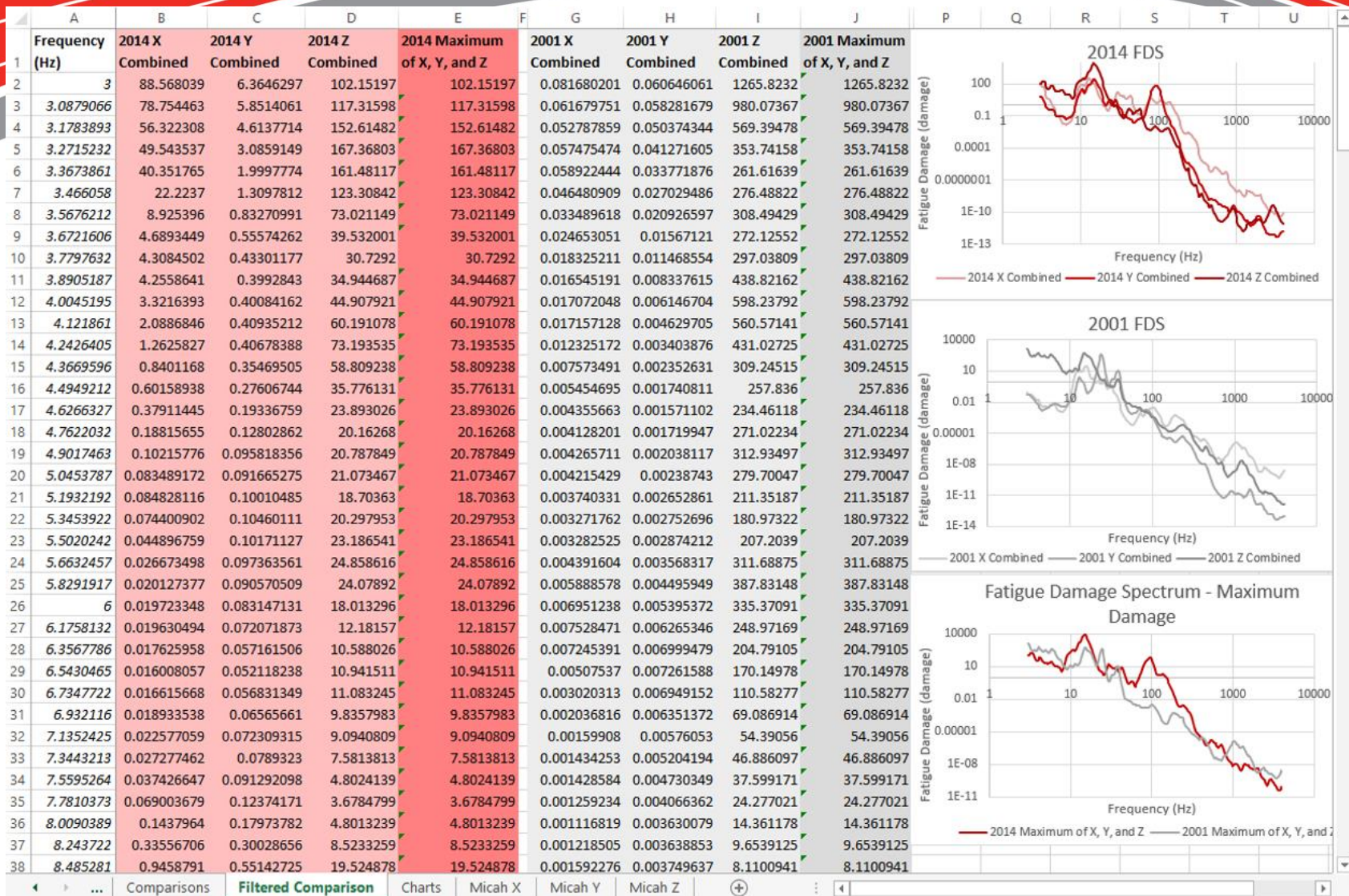


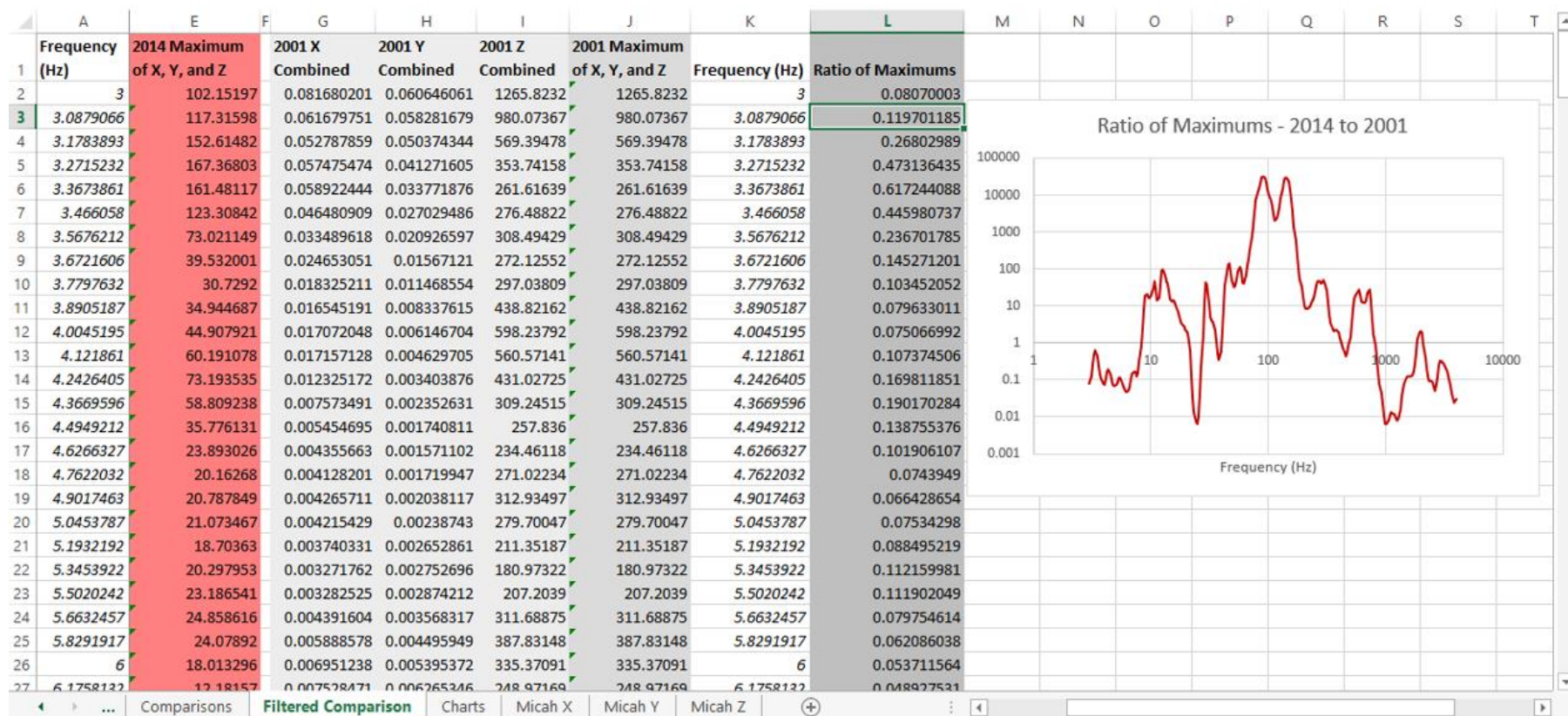
Figure 6: Perks of being an intern?



Figure 7

We set up two tri-axis accelerometers on the 2014 Mustang on the mount and on the engine computer. We were curious about the input versus response; we only ended up using the input data for this paper.





Conclusion

- FDS...
 - is a display of a waveform or waveforms' damage application to a product vs. frequency;
 - is built using the concepts of fatigue damage—primarily the S-N curve and the Miner-Palmgren rule;
 - calculates the corresponding PSD that applies the same amount of damage present in the original waveform(s);
 - facilitates target life extension and test-time reduction (test-acceleration increase);
 - can be used in tandem with Kurtosion®;
 - can combine multiple waveforms and combine their damages into one test;
 - can realistically test a product according to the product's end-use environment and time in that environment.