Examining the Dynamic Range of Your Vibration Controller

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***Dynamic Range* is one of the fundamental metrics describing the capability of a shaker controller. We all 'know' that dynamic range describes the span of small-to-large acceleration amplitude that can be properly controlled during a test. Since modern controllers are *digital* instruments, we also 'know' the dynamic range to be "six times the number of bits in the analog/digital converter"; but what do we really know? Lets examine the dynamic range of a vibration controller more scientifically.**

As shown in Figure 1, a controller forms a loop around the shaker and device under test (DUT) by providing an *analog* signal to the power amplifier driving the shaker's armature (or control valve) voice coil. This signal is called the *Drive* and the controller forms it by comparing the (analog) *Control* acceleration measured on the shaker table (or upon the DUT) with a desired *Demand* reference. The control algorithms seek to systematically minimize the difference between the *Control* and *Demand* signals by adjusting the shape and amplitude of the *Drive* signal.

It is clear from Figure 1 that the available dynamic range *of a test* is limited not only by the controller dynamic range, but also by the characteristics of the power amplifier, shaker, control-point accelerometer, test object and all the mounting hardware and methods employed. It is also clear that the controller's role involves measuring the *Control* signal, performing calculations and processes upon this measurement, and generating the *Drive* signal. The amplitude-depth of each of these three processes affects the available test dynamic range.

The shaker and its amplifier must be powerful enough to drive the DUT and fixturing to the required force and motion levels of the test. The shaker's suspension must be sufficiently linear to accurately reproduce the low amplitude details of the *Demand* and the amplifier's noise floor must be low enough not to mask these features. The accelerometer and its signal conditioning must have adequate dynamic range to follow the *Control* motion with fidelity and this range must be properly matched to the acceleration span of the test. When all of these conditions are met, the controller becomes the limiting factor in the loop's performance. Hence, it is important to have a controller with as much dynamic range as possible.

You can *never* have too much dynamic range in *any* piece of test equipment. Physical realities of a test always "stack up" unfavorably to consume every ounce of available dynamic range a system can muster. It is also important that you understand the dynamic range of your equipment, so that you can accurately predict test system performance analytically. This avoids employing time and laboratory assets on a "wing and a prayer" basis, always a costly operating philosophy.

dB(SNR): What Dynamic Range Really Says

Dynamic Range is the ratio of the largest and smallest signals that can be simultaneously properly processed by an instrument or system. This ratio is normally expressed in *decibels* (dB) to compact the range of numbers we need to think about. The largest signal is the *Full-Scale* input or output, while the smallest is the *Noise-Floor* or resolution limit of the process. Thus the dynamic range is really a signal-to-noise ratio (SNR) expressed in dB. Quite often, physics demands that the full-scale and noise-floor specifications be presented in a dimensionally-inconsistant manner. It is common to know the full-scale as a peak value and the noise floor as a root-mean-square (RMS) value. Consider the accelerometer specifications shown in Figure 2 as an example.

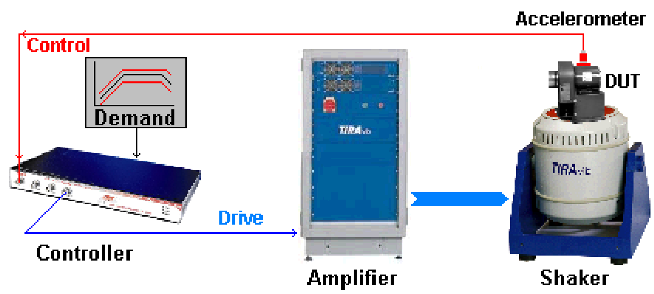
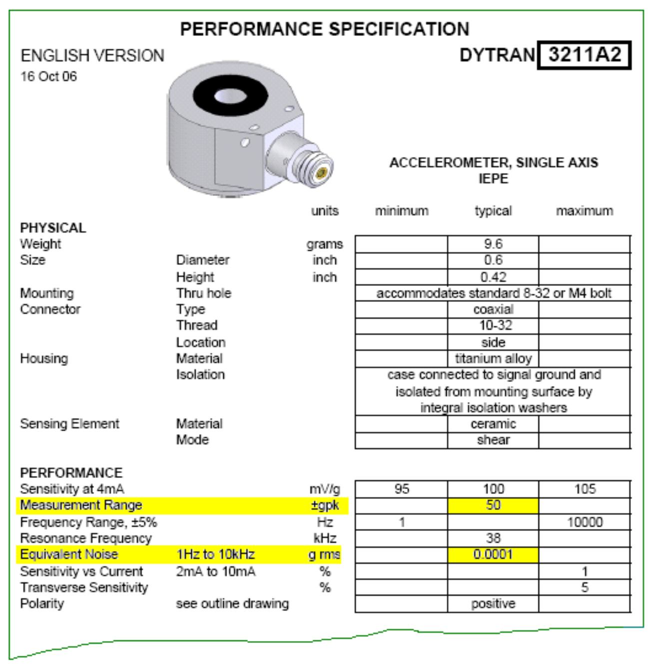


*Figure 1: A vibration controller generates a **Drive** signal that causes the measured **Control** acceleration to closely match a prescribed **Demand**.*



PERFORMANCE SPECIFICATION

ENGLISH VERSION — DYTRAN 3211A2

16 Oct 06

ACCELEROMETER, SINGLE AXIS
IEPE

		units	minimum	typical	maximum
PHYSICAL					
Weight		grams		9.6	
Size	Diameter	inch		0.6	
	Height	inch		0.42	
Mounting	Thru hole		accommodates standard 8-32 or M4 bolt		
Connector	Type			coaxial	
	Thread			10-32	
	Location			side	
Housing	Material			titanium alloy	
	Isolation		case connected to signal ground and isolated from mounting surface by integral isolation washers		
Sensing Element	Material			ceramic	
	Mode			shear	
PERFORMANCE					
Sensitivity at 4mA		mV/g	95	100	105
Measurement Range		±gpk		50	
Frequency Range, ±5%		Hz	1		10000
Resonance Frequency		kHz		38	
Equivalent Noise	1Hz to 10kHz	g rms		0.0001	
Sensitivity vs Current	2mA to 10mA	%			1
Transverse Sensitivity		%			5
Polarity	see outline drawing			positive	

Figure 2: Typical specifications of a modern accelerometer (courtesy Dytran Instruments).

This 100 mV/g sensor has a “Measurement Range” or full-scale of ±50 g and an “Equivalent Noise” or noise-floor of 0.0001 g RMS. The full-scale reflects the largest *peak* acceleration that can be measured without ‘clipping’ or limiting the ouput (voltage) signal. In contrast, the sensor’s noise-floor is a continuous broadband spectrum. Hence, the minimum resolvable signal is expressed as an RMS value over a specified bandwidth (1 Hz to 10 kHz in this case). Note that this noise-floor likely changes ‘height’ with frequency; if it were flat, it could be expressed as a constant g/√Hz *spectral density* or g^2/Hz *power spectral density*.

Dynamic range is always expressed as a ratio of *consistant* numbers. Hence, we will assume the ±50 g full-scale signal to be a sinewave and use its RMS value (0.707*50 g_{peak} = 35.4 g_{rms}) in the dynamic range calculation. This gives us a full-scale to noise-floor ratio of 35.4 divided by 0.0001 or 353,500; converting this number to decibels yields **111 dB** (= 20*$\log_{10}$[353,500]). Note that a sinewave is always used for such peak-to-RMS conversion by industry convention. Also note that the 111 dB dynamic range is for the frequency span 1 Hz to 10 kHz. If the sensor’s input is confined to a narrower bandwidth, the dynamic range will actually *increase*, reflecting integration of the noise-floor over a smaller frequency interval. The same characteristic is found in vibration controllers.

Testing Your Controller

To test the dynamic range of a device it is necessary to use a carefully constructed signal which has a peak value just below the maximum level that the device can measure, and also has a small signal that is just above the noise floor. These two signal levels could be at the same frequency and applied sequentially, or else could be two simultaneous signals applied at different frequencies. The dynamic range measurement is the ratio between the largest signal that can be measured, and the smallest that can be measured.

No single test can fully tax and capture all aspects of a controller’s dynamic range. However, the following group of tests collectively document a particular intrument’s capabilities. Each test has strengths and weaknesses which will be discussed. Some of these tests require external equipment; such tests must be approached with some trepidation. It is all too easy to lay-off a frailty of the inexperienced operator or an external instrument on the controller under examination. Digital storage oscilloscopes typically have 8-bit resolution, and functions generators rarely are harmonicly pure to -100 dB. Modern vibration controllers are as accurate as most digital signal analyzers, and often will be the most accurate function generator and signal analyzer in your lab.

We will use tests that focus on three aspects of the controller: input channels, output channels, and control algorithms. These three aspects will be interdependant, especially the control algorithm tests which depend on the signals from the input and output channels, and therefore can be no better than either of those. In many cases both the controller input and output will be used simultaneously.

We will look at two different tests of an input channel, wherein carefully produced external analog signals are applied to the *Control* input and the controller is used to capture the signal for digital analysis. Then we will examine the dynamic range of the *Drive* output by having the controller generate a full-scale fixed-frequency sinewave and measure this with an external signal analyzer. Next we will use a severe *Demand* spectrum in a “bare-wire” or “loop-back” test to examine both the *Drive* and *Control* signals, simultaneously. Two additional “loop-back” test will be discussed, one using swept sine and the other narrow bands of random noise. Finally, we will examine controlling a “high-Q” active filter acting as a simulation of a structure on a shaker.

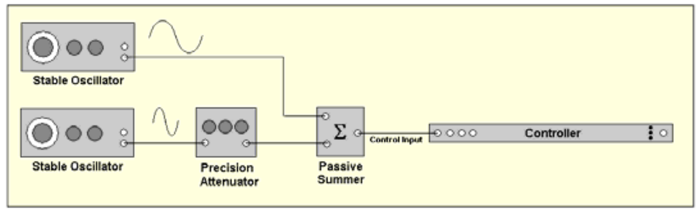


Figure 3: Testing the dynamic range of a ***Control*** *input using the two-tone method.*

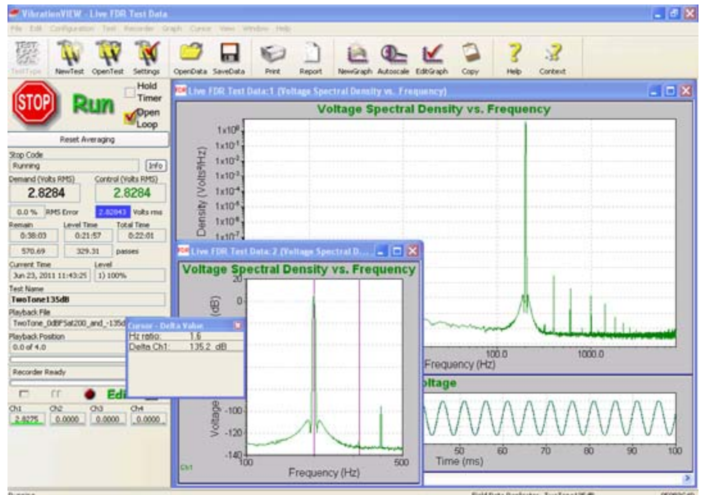


Figure 4: Two-tone test of VR9500 controller indicates 135 dB between full scale and small tone.

Two-Tone Input Test

The two-tone dynamic range of a *Control* input channel is borrowed from the IEEE recommended practice for testing a time-compression or FFT spectrum analyzer. As shown in Figure 3, an analog signal with two sinewave components is applied. One component is adjusted to be a full-scale input. The second sine, at a different frequency, is attenuated until it is just ‘lost’ in the baseline of a spectrum computed from the sum. The ratio of these components is determined by the setting of a precision attenuator, and this setting is recorded as the dynamic range when the smaller signal is just indistinguishable from the spectral floor.

The test signal required for this test may be generated by a high-performance arbitrary function generator. Alternatively, two high-quality oscillators or sine synthesizers, a precise attenuator and a precision summing circuit may be used to ‘build’ the signal. The analog sine sources need to provide an output greater than the full-scale of the controller input channel, to allow use of a *passive* summing circuit. Using an active summer introduces an external noise-floor to the equation and introduces possible signal artifacts including intermodulation products.

The results of this test depend strongly on the block length used for the FFT and the type of window function used. The quantization noise of the primary tone acts as a dither signal, and this allows measurement of low amplitude signals that could not be measured independantly. For example, with a 65,536 sample block length and a hanning window function, an ideal 14-bit system would show 120 dB with this test. Obviously this is much greater than the 84 dB SNR one would normally expect from a 14-bit input. For this reason, this test is not a good measure of dynamic range.

Also, note that the frequency peaks due to harmonic distortion will in all likelihood be higher than the lower of the two summed signals. If the two signals being measured are harmonics of each other, then the level of harmonic distortion will be the limiting factor. This is evident in Figure 4, where the harmonics of the primary tone are more prominent than the low level tone. So, while the noise floor allows for distinguishing some signals 135 dB apart, this test may be more useful as a measure of the amount of harmonic distortion present. What cannot be determined from this test is whether the harmonic distortion is due to the arbitrary function generator or is due to the controller input electronics.

Effective Bits Input Test

This is a more recent test method sanctioned by the IEEE for digital recording devices. As shown in Figure 5, a single sinewave is applied to the *Control* input channel. An untriggered digital recording of the sine is made by the controller input hardware and this data is curve-fitted to yield the four parameters (**f**, **A**, **B** and **C**) of a model of the form: y(t) = **A** Cos(2π **f** n Δt) + **B** Sin(2π **f** n Δt) +**C**, where **Δt** is the known inter-sample interval and **n** is the sample count. This parameter identification allows the 'signal' to be separated from the 'noise', permiting calculation of a SNR. The result is then expressed in terms of "effective-bits", rather than in dB.

To understand the conversion from a dB(SNR) to "effective-bits", consider the perfect "**n**-bit" converter. A perfect "**n**-bit" converter exhibits $\mathbf{2^n}$ unique output codes uniformly spanning its $\pm \mathbf{V_{FS}}$ input voltage range. Each "least significant bit" (LSB) change in the output code reflects a change in input voltage by an amount, $\mathbf{V_q}$, termed the *quantization voltage*, where $\mathbf{V_q = V_{FS} / 2^{n-1}}$. Note that the code representation is *exactly* correct at only $\mathbf{2^n}$ specific voltages; in all other cases, it is in error by as much as $\pm \mathbf{V_q/2}$. Hence the converter is said to be precise to ± ½ LSB. As a result, the ideal converter has an RMS noise-floor of $\mathbf{V_q/\sqrt{12}}$. When a sine signal of $\pm \mathbf{V_{FS}}$ peak is applied to the converter, the (RMS) signal/noise ratio is given by:

$$dB(SNR) = 20\log_{10}(SNR) = 20\log_{10}\left(\frac{\frac{V_{FS}\sqrt{2}}{2}}{\frac{V_q}{\sqrt{12}}}\right) = \qquad (1)$$

$$20 \cdot \log_{10}\left(2^{n-1}\sqrt{6}\right) = 6.02n + 1.76$$

This result is plotted in Figure 6, and we now have a better understanding of the long-accepted "6 dB-per-bit" truism! Effective-bits compress a large range of numbers into a more comprehensible span, as does a dB calculation. For example, the accelerometer previously discussed has a claimed SNR of 111 dB. Use this number to enter the verticle axis of Figure 6; read the equivalent "number of bits" (slightly more than 18) from the horizontal axis. Clearly, this sensor would have no problem in feeding a 16-bit ADC to full dynamic range capability. It would be a clear 'choke-point' in a 24-bit system.

The major advantage of an effective-bits test is that it requires no comparitive measurements. The frequency of the test sine must be within the selected bandwidth of the controller. Its amplitude must be less than the $\pm \mathbf{F_s}$ voltage of the input. However, neither the frequency nor the amplitude of the test signal need be measured or known. The sine must be *stable* in frequency and amplitude and *pure* in waveform. For this reason, the *analog filtered* output of a *digital synthesizer* is the prefered signal source. However, the controller must provide access to the raw waveform data, and a digital curve-fitting algorithm is required

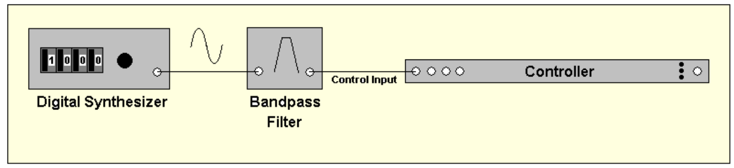


*Figure 5: Testing the dynamic range of a **Control** input using the IEEE Effective Bits Method.*

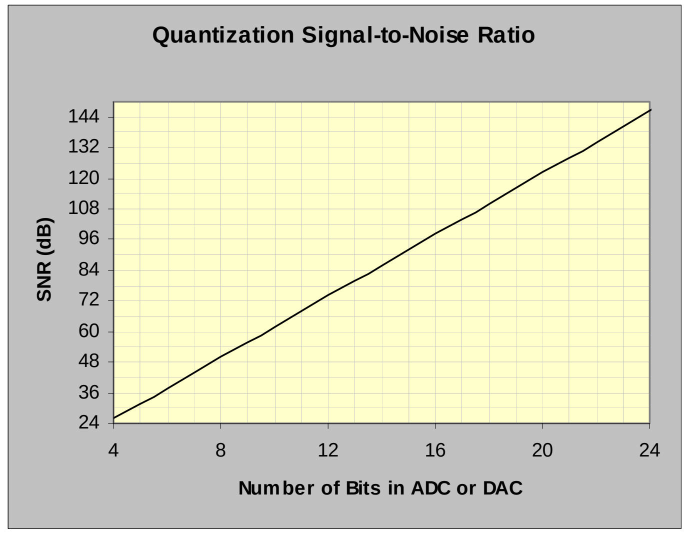


Figure 6: Relationship between effective bits and dynamic range or SNR.

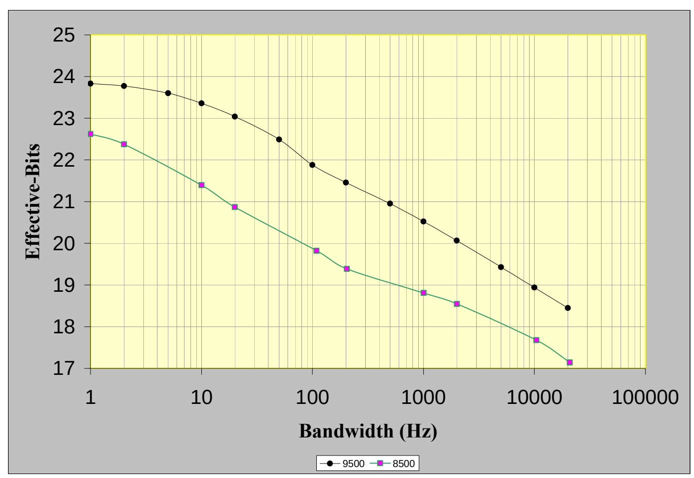


Figure 7: Measured Effective-Bits of the VR9500 and VR8500 Control input channel at various bandwidths.

to accomplish the test. The IEEE particularly warns against using an FFT as the curve-fitting process and recomends the direct application of "least-squares minimization" between the model and the acquired samples

This test was run on a Vibration Research VR9500 controller, using the controller output as a high quality sine wave synthesizer, and using the RecorderVIEW feature to record the raw input waveforms to disk. These waveforms where then loaded into MATLAB® and processed with a curve-fitting algorithm. Typical test results for the VR9500 are shown in Figure 7. The four test bandwidths of 20 Hz and less correspond to selected sine tracking filter bandwidths. The six higher bandwidths show typical broadband results; these include the 200 Hz ASTM transportation test bandwidth, the 2000 Hz NAVMAT test bandwidth and the VR9500's maximum bandwidth. The corresponding dynamic ranges for these bandwidths are shown in Figure 8.

Single-Tone Output Test

The test shown in Figure 9 looks at the the spectrum of a fixed-frequency, full-scale sinewave produced by the controller. Output dynamic range is recorded as the dB difference between the sine peak and the spectrum's noise floor. Clearly, the results can be no better than the two-tone dynamic range of the spectrum analyzer employed, so this test is typically limited by the capabilities of the analyzer rather than the controller. As with the two-tone input test, this test is also strongly influenced by the block length of the FFT used in the analyzer, and therefore will not correlate well with the true dynamic range of the controller.

Figure 10 shows the spectrum of a 1000 Hz ±2 Volt sine generated by a VR9500 controller output, and analyzed using an external FFT analyzer. The background noise is more than 126 dB below the signal, with the highest harmonic peak about 110 dB below the signal.

Loop-Back tests

The drawback of the preceding tests is the requirement for external equipment and/or external mathematical analysis. However, vibration controllers have the ability both to produce output and analyze the input, and both of these functions are used simultaneously while in operation. Therefore it is desirable to apply a test, which uses the controller's output and input, so that both are tested simultaneously, and no additional equipment is required. The results of the test will then reflect the combined dynamic range of both the outputs and inputs. It is also possible to do these same tests with only the output, using an external signal analyzer, or only the input using an external signal synthesizer. This would allow separate results to be stated for both the output and the input, at the expense of requiring additional equipment.

Sine Loop-Back Test

In this test, the *Drive* is looped back to the *Control* input as shown by Figure 11. A Swept-sine test is run, with the test specifying a fixed frequency sine with an exponential decrease in amplitude over the duration of the test. The amplitude starts at the full-scale level and then decreases to a level below the anticipated dynamic range of the controller. The *Drive* and *Control* are viewed as RMS-versus-time plots with a log amplitude axis. The *Transmissability* (RMS-to-RMS ratio) of the *Control* with respect to the *Drive* is also displayed. As shown in Figure 15, both of these traces are clean straight lines within the dynamic range of the controller. The amplitude is at the controller's full-scale at the left side of the display and sweeps down "into the noise" on the right. In this region, the *Transmissibility* is exactly **1.0** (for 1000 *mV/g*). At the (right) edge of the dynamic range, where the envelope of the *Transmissibility* spans **.707** to **1.41**, the signal and noise values are at the same amplitude. The *Drive* voltage RMS at this point in the test is equal to the instrument's RMS noise floor within the frequency band passed by the tracking filter. The amplitude of the drive voltage at this point is therefore a measure of the noise floor. The dynamic range of the output and input is the ratio of the maximum voltage (full scale voltage) to this noise floor measurement.

It is important to note here that the RMS noise floor in this test is the amount of noise measured *after* the tracking filter. The narrower the tracking filter, the less noise it will pass through. Therefore the sine tracking filter bandwidth is an important parameter in this test, and

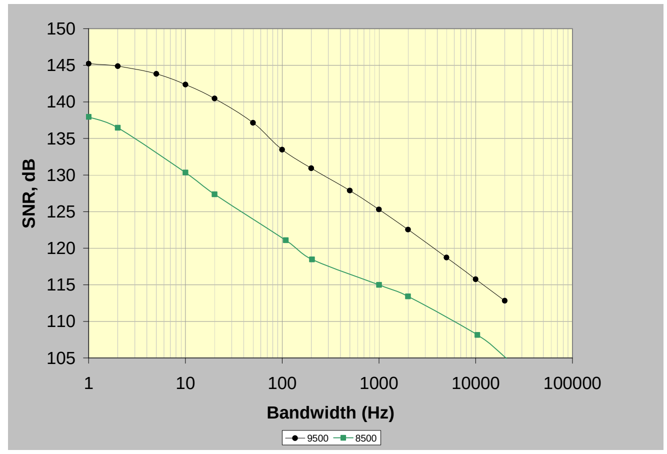


Figure 8:Dynamic range of the VR9500 and VR8500 controller at various bandwidths.

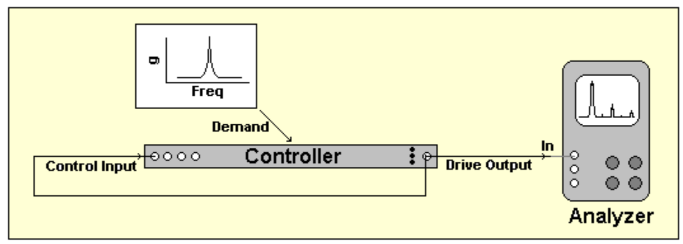


*Figure 9: Testing the dynamic range of the **Drive** output.*

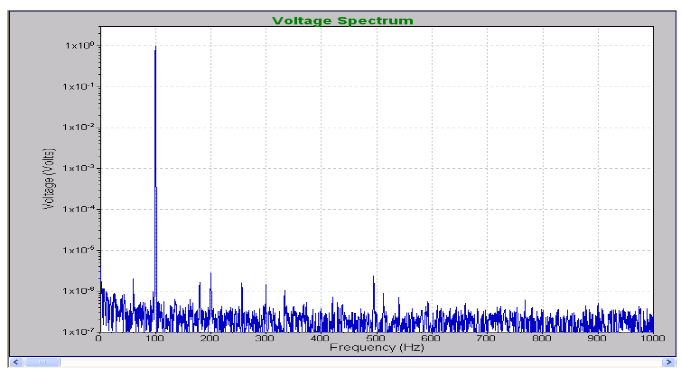


Figure 10: This single-tone test of the VR9500 Drive output shows roughly 110 dB dynamic range.

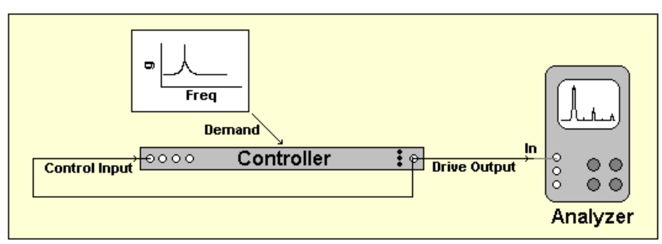


Figure 11: Ramped-sine loop-back test setup.

when using results of this test to compare controllers, it is important to use the same bandwidth for each controller. Figure 12 illustrates that the VR9500 controller, with a 10 Hz tracking filter bandwidth, shows a dynamic range of more than 130 dB.

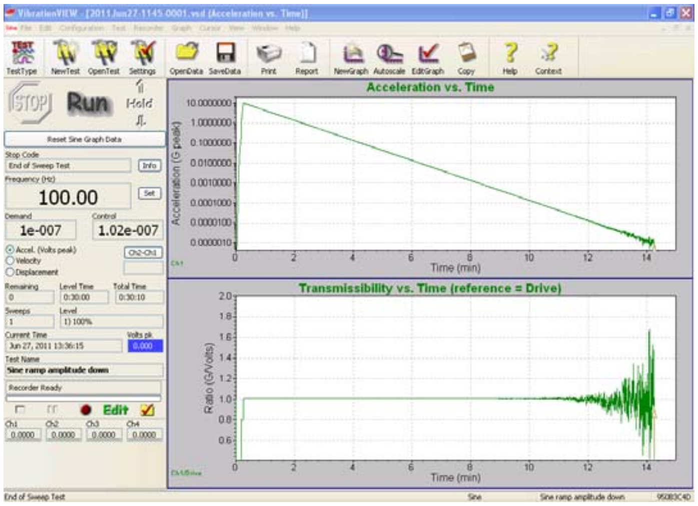


Figure 12: Ramped-sine test of the VR9500 controller indicates about 130 dB input/output dynamic range.

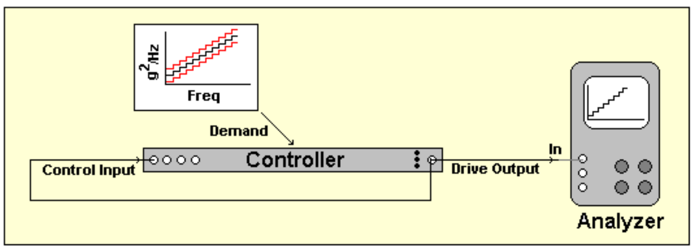


Figure 13: Narrow-band random loop-back test setup.

Random Loop-Back Test

The controller can be tested in a similar manner using a Random test, employing a sequence of stepped demand levels ranging from below the noise floor to the full scale spectral density as shown in Figure 13. To determine the lower limit the *Coherence* between the *Drive* and *Control* is computed. The lower limit is the level prior to the point where the coherence drops below 0.5. The dynamic range measurement is then the ratio of the full scale density over this lower limit.

The full scale density for this test is determined by the level where the RMS of the highest band nearly makes the signal peaks reach the full scale voltage. This will happen when $\sqrt{(\text{Density}_{max}*\text{Bandwidth})} = V_{FS}/6$ where V_{FS} is the full-scale voltage level, and Bandwidth is the frequency width of the steps used. The implication of this is the full scale spectral density will depend on the bandwidth used in this test, and as a result, the resulting dynamic range measurement will depend on the bandwidth used in the test.

The narrower the bandwidth used in the test, the higher the maximum density level that can be achieved, and therefore the wider the dynamic range value that will be reported. Because of this, when comparing controllers using this test one must be careful to use the same parameters on both controllers.

Figure 14 shows a loop-back random test being run on a VR9500 controller. As shown, 24 *Demand* PSD steps, each 50 Hz wide, span 115 dB. Control appears to be tight from 3.16 x 10^{-2} V^2/Hz down down to 1 x 10^{-13} V^2/Hz, a span of 115 dB.

However, the accompanying Coherence and Transmissability plots indicate that the noise floor is closer to 3.16 x 10^{-13} V^2/Hz for a conservative Random Dyamic Range of 105 dB. Note that the Coherence is greater than **0.9** all the way down to –105 dB. It is still about **.8** at –110 dB where the Transmissability departs from **1.0**.

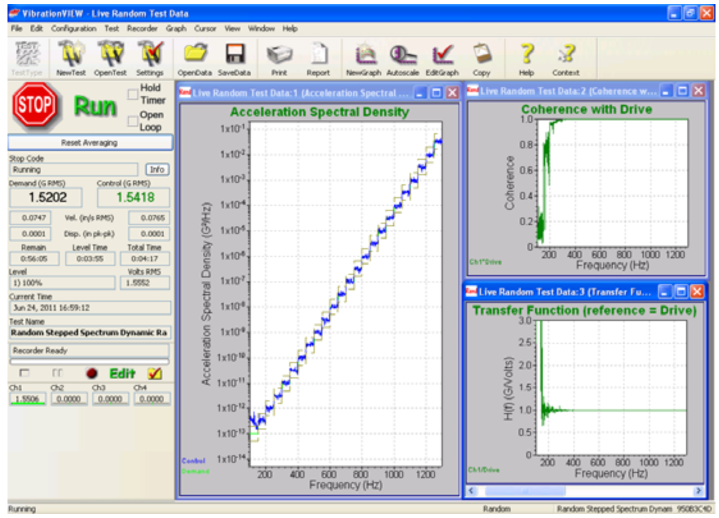


Figure 14: Random test indicates 105 dB of clean dynamic range for the VR9500.

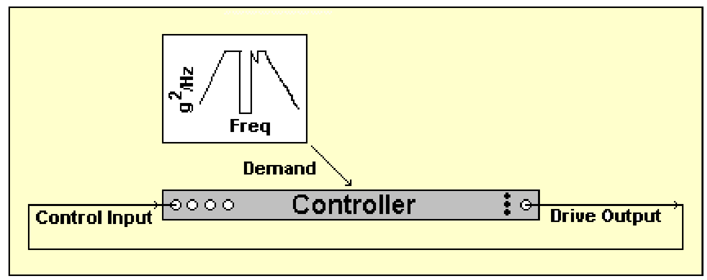


Figure 15: Testing both **Drive** and **Control** dynamic range using a severe **Demand**.

Note that this test is run with a *Demand* PSD that has increasing amplitude steps with frequency. This assures that the lowest amplitude steps in the *Control* signal reflect the instrument's *noise-floor* rather than *harmonic distortion*. That is, no step of this signal introduces significant harmonics of itself in other steps.

Input/Output Test Using a Severe Demand

An interesting alternative to the previous tests was developed by the Chinese government around 1988. This test (see Figure 15) is meritorious in that it requires *no* external equipment; it taxes both the *Control* input and *Drive* output simulatneously. It requires the testor to understand nothing beyond the operation of the controller and does not involve curve-fitting or advanced mathematics. In short, it is absolutely elegant in its simplicity.

The key to this efficient evaluation is the frequency span between **350** and **500 Hz**, as shown in Figure 15. The *Demand* spectrum in this region is sucessively programmed to lower g^2/Hz power spectral density levels until the controller fails to conduct the test. The lowest 'notch' level at which control can be achieved determines the *control system* dynamic range.

As shown in Figure 16, the VR9500 sucessfully controlled the notch down to a depth of –100 dB with respect to the highest PSD level. It could not follow the –110 dB demand of the lowest figure. Figure 17 shows the 110 dB test in more detail, including the limits and the *Drive* signal.

Note that the JJG 529-88 test profile is specified in g^2/Hz units. Also recall that to test dynamic range the signals used must be carefully crafted such that the peak levels are just below the maximum value the system is capable of measuring. Therefore the operator must select the mV/g sensitivity of the (non-existant) accelerometer such that the peak levels of the 10 g_{RMS} signal are just below the maximum input voltage of the system, for the input range being tests. This allows this test to be independent of the input range used. For a system with ±10 Volt *Control* and *Drive* full-scales, this optimum is about 100 mV/g, which results in voltage levels of 1 V_{RMS}, and 6 V_{PEAK} At this point the achievable depth of the notch will measure the range between the full scale signal and the noise floor. If the sensitivity is increased above 200 mV/G then the waveform peaks will begin to saturate. This will be evident as a sudden increase in the measured G^2/Hz level in the 350-500Hz notch, and will severely impact the test results.

Now if we factor out the sensitivity scale factor and consider the relationship between the maximum V^2/Hz level and full scale input voltage, and also the relationship between the minimum V^2/Hz level and the noise floor, we can get some more insight into this test. For the given spectrum, the maximum PSD level is 10^{-3} times the square of the RMS voltage, V_{RMS}. Assuming a crest factor of 5, the maximum achievable V_{RMS} level will be $V_{FS}/5$. The minimum achievable V^2/Hz level, assuming a flat noise spectrum, will be $(V_{noise}{}^2)/(2000\text{ Hz})$, where V_{noise} is the RMS noise floor. The expected random dynamic range (RDR) of the JJG 529-88 test will then be the dB ratio of these two power spectral densities.

$$dB(RDR) = 10\log_{10}\left(\frac{10^{-3}V_{RMS}^2}{V_{noise}^2/2000}\right) = 10\log_{10}\left(\frac{2\cdot V_{RMS}^2}{V_{noise}^2}\right) = \qquad (2)$$

$$10\log_{10}\left(\frac{2\cdot (V_{FS}/5)^2}{V_{noise}^2}\right) = 10\log_{10}\left(\frac{V_{FS}\sqrt{2}}{5\cdot V_{Noise}}\right)^2$$

This result can be recast in terms of the SNR measured in an *effective-bits* test. V_{Noise} can be recognized as including the quantization noise of an 'ideal' converter and the noise of the supporting signal conditioning. Therefore the RDR can be restated in terms of SNR as follows.

$$dB(RDR) = 20\log_{10}\left(\frac{V_{FS}\sqrt{2}}{5\cdot V_{Noise}}\right) = \qquad (3)$$

$$20\log_{10}\left(0.4\left(\frac{V_{FS}\sqrt{2}/2}{V_{Noise}}\right)\right) = dB(SNR) - 7.96$$

This is a very interesting result, because it tells us that the best results one could expect from this test will be 8 dB less than the dB(SNR) as measured using a sine tone. This is to be expected because the peaks in a random signal are higher than for a sine signal, so the random full-scale RMS will be lower than the sine full-scale RMS.

To apply these results practically, we refer to Figure 8 and note that for the VR9500 with a 2000Hz bandwidth, the SNR is 122 dB, so we could reasonably expect a 114 dB result from the the JJ 529-88 test. But we only found a result of 100 dB. What accounts for this difference? The answer lies in the high band from 80 to 300Hz. The notch from 350 to 500Hz corresponds to the harmonics of the 80 to 300Hz band. Any harmonic distortion of the 80 to 300Hz band will

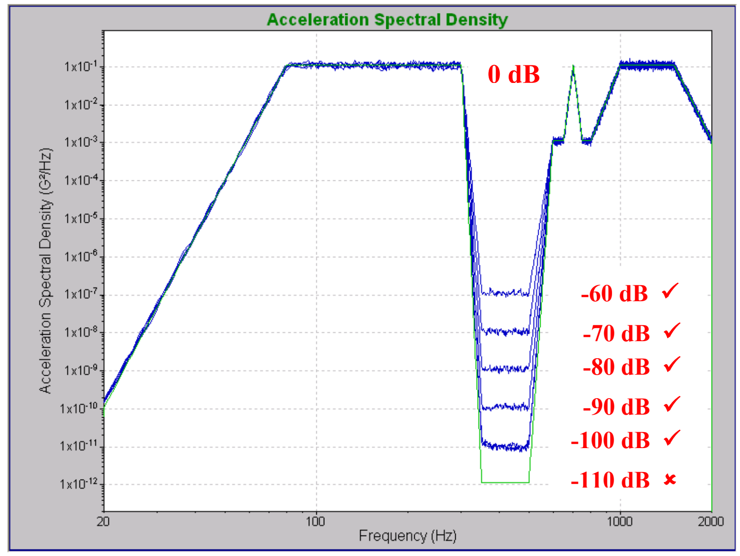


Figure 16: VR9500 performing the Chinese JJG 529-88 challenge test at 60 through 110 dB.

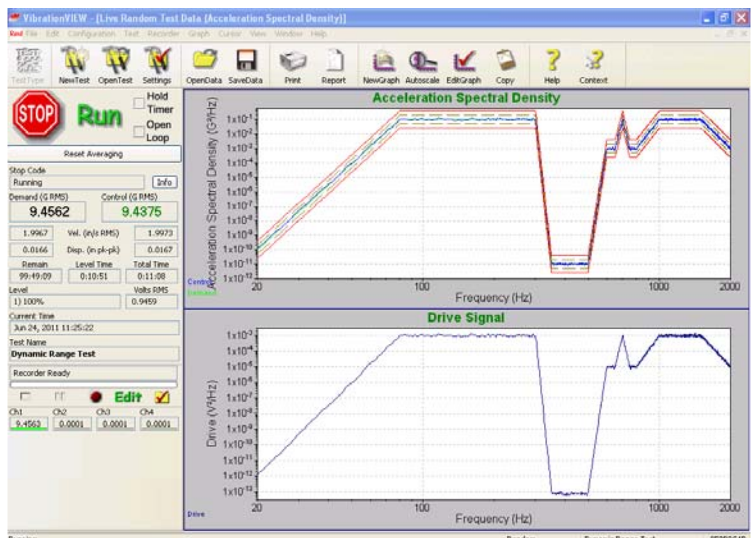


*Figure 17: Details **of JJG 529-88** operation at 100 dB for VR9500 showing Control with Limits and Drive signal*

show up as an increased noise level in the 350 to 500Hz notch. Therefore this test measures dynamic range up to a point (typically to 16 effective bits), but beyond that point the results are limited by *harmonic distortion*. Since most current controllers have more than 16 bits of resolution, their results using this test will be limited by the harmonic distortion of the system, and not by the dynamic range.

Random Algorithm and Output Test Using a "high-Q" Filter to Simulate a Shaker and Structure

There is one valid criticism that can be directed at the JJ 529-88 test: it does not demonstrate control over any sharp resonance peaks and notches that may exist in a structure. Our next test addresses this issue. At least two controller manufacturers have built active "challenge filters" and issued them to their sales staff for field demonstration purpose. These two designs are remarkably similar; both manufacturers selected similar peak and notch frequencies and amplitudes. The

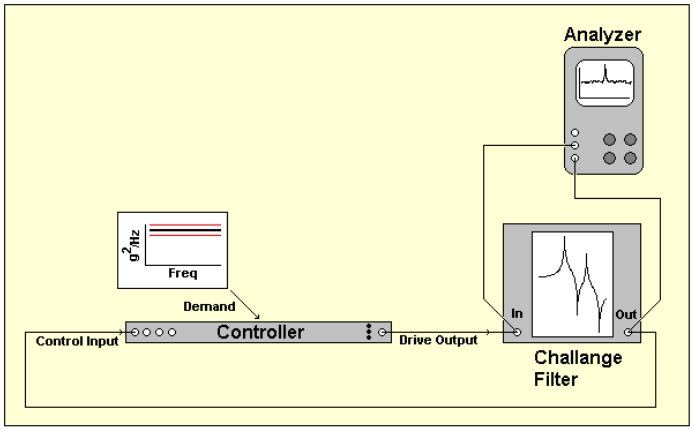


Figure 18: Testing dynamic range by closing the loop around a simulation filter.

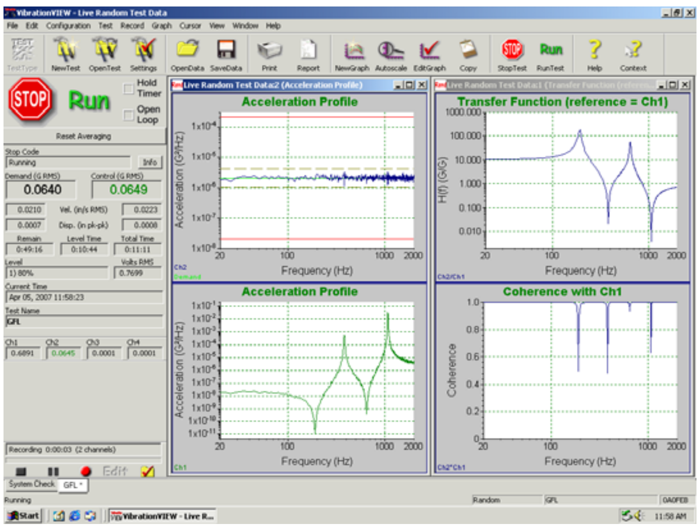


Figure 19: Closing the loop around a challenge filter with the VR 8500 using 800 lines of resolution.

VR9500 was recently tested against the newer of these two challenge filters.

In this test, the filter is driven by the *Drive* signal and its ouput is applied to the *Control* channel, as shown in Figure 18. The test *Demand* is a flat spectrum spanning 20 Hz to 2000 Hz. Thus, when fully-controlled, all of the significant signal dynamics will be reflected by the *Drive* signal. The implication of this is the test exercises the control algorithm and the output signal, but since the reference spectrum is flat, the input dynamic range is essentially untaxed by this test.

Testing these filters is a challenge until a couple of things are understood. Firstly, the filter uses active electronics with a limited voltage range, The filter becomes wildly non-linear when it is overloaded. Neither design incorporates any form of overload indication, so the experimenter needs to determine the maximum 'white' drive that can be applied without causing the filter to grossly misbehave.

Secondly, the filter's DC gain of 10 (20 dB) is a bit deceptive. As it would suggest, the RMS output of the filter is greater than the RMS input, *when the input frequency is below the frequency of the first peak*. However, when a *white input* is applied to the filter, the output RMS is

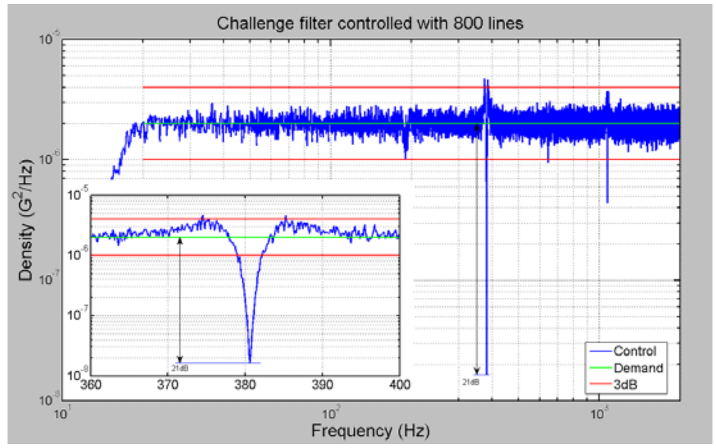


Figure 20: 800-line Control signal spectrum-analyzed by an external analyzer with more resolution.

about 36 dB *greater* than the input RMS. Then, when the input is properly 'shaped' to produce a *white filter output*, the output RMS is about 23 dB *less* than the input RMS. (See *RMS Gain* sidebar for the explanation.)

All of this means that the test *must* be conducted with a properly chosen RMS demand level. The 20 dB DC gain might lead one to think the reference spectrum should be chosen to give $1V_{RMS}$. But once the drive signal is properly shaped to make the filter output *white*, the filter input would be 23 dB higher than $1V_{RMS}$, or $14.6V_{RMS}$. Clearly this would overdrive the filter input! The proper choice of the demand level is found by determining the input level that overloads the filter and then setting the reference spectrum to be 23 dB lower than this level. Typically such a filter will work properly up to a $1V_{RMS}$ input, so setting the reference spectrum to give $68mV_{RMS}$ would yield the best results.

Figure 19 illustrates successful control-lock on a challenge filter using a VR 8500 with 800 lines of control, with all *Control* lines within ±3 dB of the *Demand* line. From this figure one could be led to believe that the controller has passed this test. However the challenge filter has a notch at 380 Hz with a bandwidth of only 0.35 Hz. The control bandwidth of 2000Hz divided by 800 lines of control gives a controller line resolution of 2.5Hz. Knowing this, we recognize it is impossible for *any* controller to properly control this filter with only 800 lines!

Adding an external analyzer to monitor the Drive and Control signals is probably prudent for independent validation of any loop-back test. However in this particular case we see that depending entirely upon an instrument to analyze its own worth is a serious error in judgement. In lieu of a stand-alone analyzer, you might substitute the use of a digital recorder (or recording software using the controller's hardware) and off-line analysis using software such as MATLAB®.

Figure 20 shows the *Control* signal as analyzed independently using a high-resolution FFT, with line resolution of 0.08Hz. When analyzed independently we can see that the *Control* signal actually has more than 20 dB of error around the notch, far beyond the 3 dB the controller display reveals! From this we can see that any challenge filter test *must* be verified using an *independent* analyzer. Simply relying on the controller's display allows the controller imperfections to hide the true errors present in the signal.

To properly control this test we need a line resolution of 0.35Hz or better. This can be achieved by increasing the number of lines resolution employed by the controller. Figure 21 illustrates a successful control-lock on a challenge filter using a VR 8500 with 13,000 lines of control, with all *Control* lines within ±3 dB of the *Demand* line. Figure

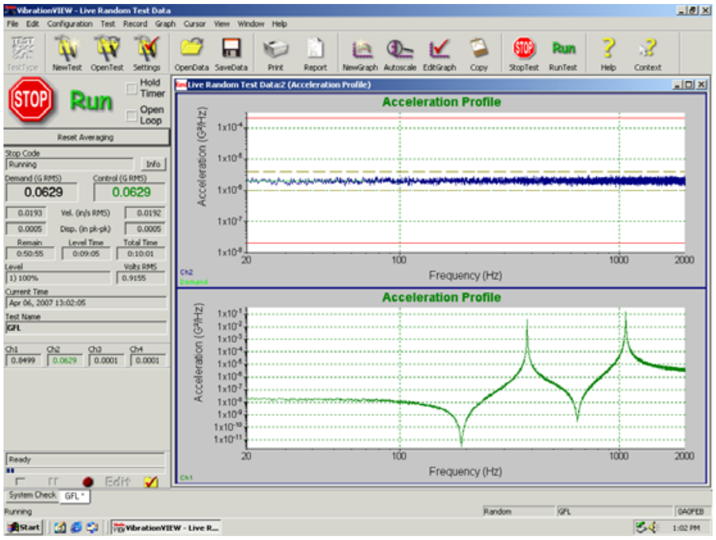


Figure 21: Closing the loop around the challenge filter with 13,000 lines of resolution.

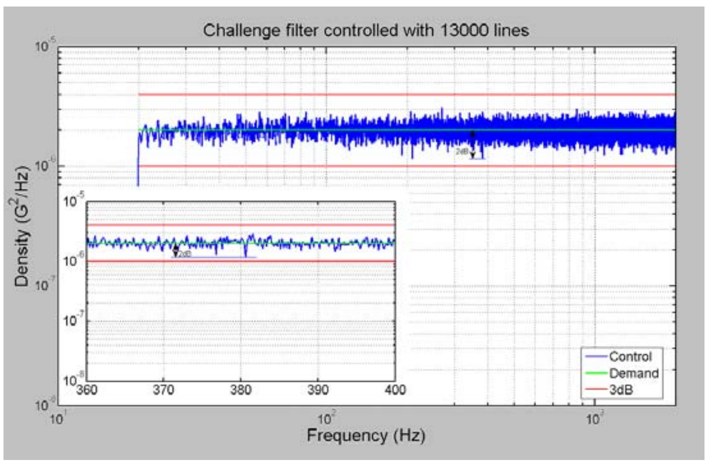


Figure 22: 13,000-line Control signal spectrum-analyzed by an external analyzer.

22 shows the independent analysis of this test, demonstrating proper control of this filter using 13,000 lines, with a maximum error of less than 2 dB.

Figure 23 presents a summary of worst-case loop-error as a function of control resolution. In all cases, the worst error occurred at the 380 Hz filter notch. This investigation disclosed that at least 8,000 lines of controller resolution are required to control this challenge filter within ±3 dB over the 20-2000 Hz NAVMAT bandwidth.

Figure 24 presents the Transfer and Coherence Functions of the filter from the test of Figure 21. The Dynamic range of the test is read from the amplitude-extremes of the Transfer Function. The high Coherence values at each Transfer Function peak and valley indicates 'clean' control and linear filter behavior.

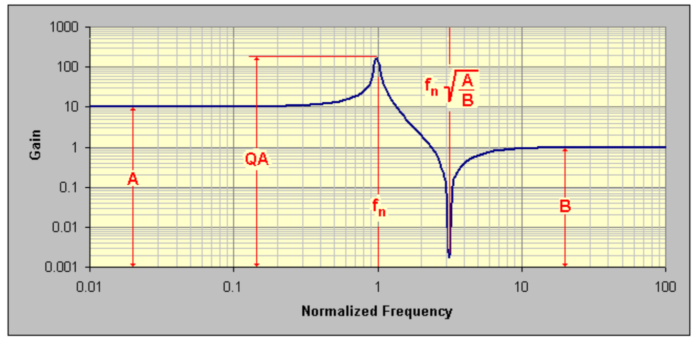
Proponents of this test claim the dynamic range of the controller can be determined by taking the ratio of maximum drive output to the minimum drive output. In the above test we see the VR9500 demonstrates greater than 120 dB in this test, which corresponds to the dynamic range of the challenge filter. However this test isn't truly testing dynamic range.

In fact, a 16-bit controller with 13000 lines can also successfully demonstrate greater than 100 dB of "dynamic range" using this test, even though the true dynamic range of a perfect 16-bit converter is only 96 dB. To understand how this can be, consider that when running the challenge filter test the drive output from the controller must be the inverse of the filter transfer function to get a flat spectrum output from the filter. The notch in the filter transfer function will correspond to a very narrow peak in the required drive output. The maximum value of that peak is measured in V^2/Hz, where the Hz portion is determined by the bandwidth of the peak. It follows that the maximum V^2/Hz value can be increased *without increasing the signal level* simply by reducing the bandwidth.

In the sample case examined here the highest peak has a 2.5 Hz bandwidth. If we assume a 1 V_{RMS} signal with half of the total signal concentrated in this peak, then we would expect the V^2/Hz level to be

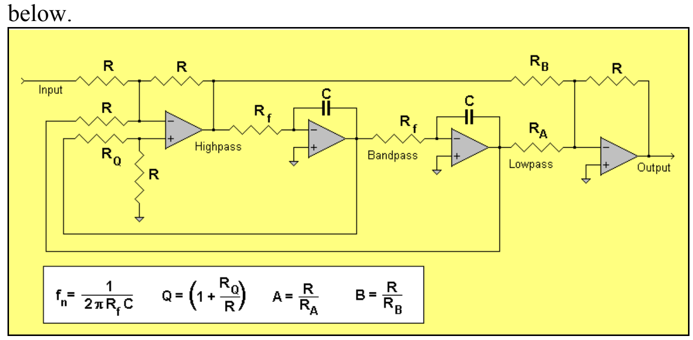
Challenge Filter

Two controller manufacturers have produced extremely similar "challenge filters", each based on two cascaded four amplifier state-variable filter sections. Each section sums a highpass and lowpass component in the manner normally used to form a n*otch filter*. However, the summing gains of the final stage are deliberately unequal. This produces a high peak at the tuned frequency, f_n, and a deep notch (a 'zero') at a second frequency determined by the final gain ratio.



Gain characteristics of each section of challenge filter.

The Transfer Function of the filter section is shown above with certain key features noted. The circuit producing this Transfer Function is shown below. Five component values (C, R_f, R_Q, R_A and R_B) must be chosen to tune this circuit. The simple tuning equations are shown below.



Simplified schematic of filter section; the filter is comprised of two differently tuned sections in series.

RMS Gain

It is interesting to observe the RMS values of the *Drive* and *Control* signals during the controller's pretest equalization interval. As shown below, the *Drive* RMS (black) makes a gradual monotonic increase as the controller converges on the proper $H_{AB}^{-1}(f)$ to equalize the challenge filter's transfer function, $H_{AB}(f)$. The *Control* RMS (red) increases rapidly at the start, and then soon levels out.

The ratio of these two RMS values (blue) is particularly interesting. Note that the filter's output RMS (*Control*) is considerably larger than the filter's input RMS (*Drive*) when the equalization process starts. At this time, the *Drive* is essentially a 'white' noise, reflecting the constant-amplitude *Reference* entered for the challenge. The early *Control* spectrum shape looks like the filter's gain characteristic, $|H_{AB}(f)|$. At this time, the filter's output RMS is a large multiple of the input RMS.

After the *Drive* and *Control* signals have converged on their stable run values, the *Control* spectrum is flat, while the *Drive* spectrum looks like the reciprocal of the filter's transfer function gain, $|H_{AB}^{-1}(f)|$. At this time, the filter's output RMS is only a small fraction of the input RMS.

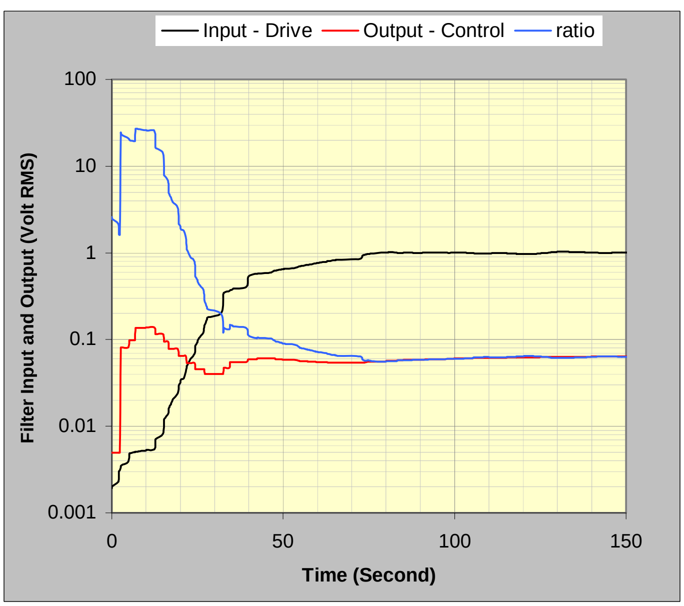
This effect is entirely normal and it is predictable from the filter's *Transfer Function*, $H_{AB}(f)$. The following figure shows $H_{AB}(f)$ in black and its inverse, $H_{AB}^{-1}(f)$ in red.

We know that the magnitude of a transfer function, $|H_{AB}(f)|$, relates its input power spectral density, $G_{AA}(f)$, and its output power spectral density (PSD), $G_{BB}(f)$, in accordance with:

$$G_{BB}(f) = |H_{AB}(f)|^2 \bullet G_{AA}(f) \tag{4}$$

The RMS value (over a bandwidth from f_1 to f_2) of a signal relates to the PSD, specifically:

$$RMS_x = \sqrt{\int_{f_1}^{f_2} G_{xx}(f)\, df} \tag{5}$$



If the filter is excited by white noise of constant power spectral density, δ (volt²/Hz), it has an input RMS value of:

$$RMS_A = \sqrt{\int_{f_1}^{f_2} G_{AA}(f)\, df} = \sqrt{\int_{f_1}^{f_2} \delta\, df} = \sqrt{\delta(f_2 - f_1)} \tag{6}$$

The corresponding output RMS value is:

$$RMS_B = \sqrt{\int_{f_1}^{f_2} G_{BB}(f)\, df} = \sqrt{\int_{f_1}^{f_2} |H_{AB}(f)|^2 G_{AA}(f)\, df} = \sqrt{\delta \int_{f_1}^{f_2} |H_{AB}(f)|^2\, df} \tag{7}$$

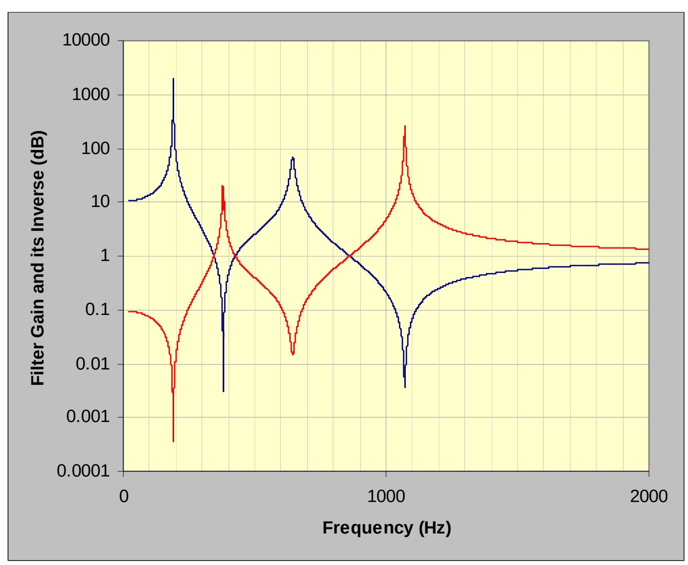
This allows the ratio of RMS values to be stated:

$$\frac{RMS_B}{RMS_A} = \sqrt{\frac{\int_{f_1}^{f_2} |H_{AB}(f)|^2\, df}{(f_2 - f_1)}} \tag{8}$$

For the challenge filter examined, the ratio of equation (10) evaluates to 62.9 (or 36 dB). Hence, the output is considerably greater than the input at the initiation of control equalization.

Once the *Control* signal has been forced to match the required flat spectrum, the situation changes. With a flat filter output (*Control*) spectrum, we can evaluate the output/input RMS ratio by computing $H_{AB}^{-1}(f)$ and substituting it into equation (9). This results in a final *Control/Drive* ratio of .0679 or –23 dB. That is, the *Control* RMS is considerably *less than* the *Drive* signal, once control to a 'white' spectrum is achieved.

$$\frac{RMS_B}{RMS_A} = \sqrt{\frac{(f_2 - f_1)}{\int_{f_1}^{f_2} |H_{AB}^{-1}(f)|^2\, df}} \tag{9}$$



$(0.5\ V)^2/(2.5\ Hz) = 0.1\ V^2/Hz$. Indeed, when scaled to a 1 V_{RMS} level, we find the drive spectrum has a peak of $1 \times 10^{-1}\ V^2/Hz$. On the lower end, a 16-bit converter with ±10V range has a quantization noise floor of $8.8 \times 10^{-5}\ V_{RMS}$, or $3.1 \times 10^{-12}\ V^2/Hz$. Taking the ratio of these two numbers, we see a 105 dB range can be shown with this test on a 16-bit system that we already know cannot exceed 96 dB of true dynamic range! For this reason the challenge filter test, while useful to exercise the control loop and line resolution, should not be used as a measure of dynamic range.

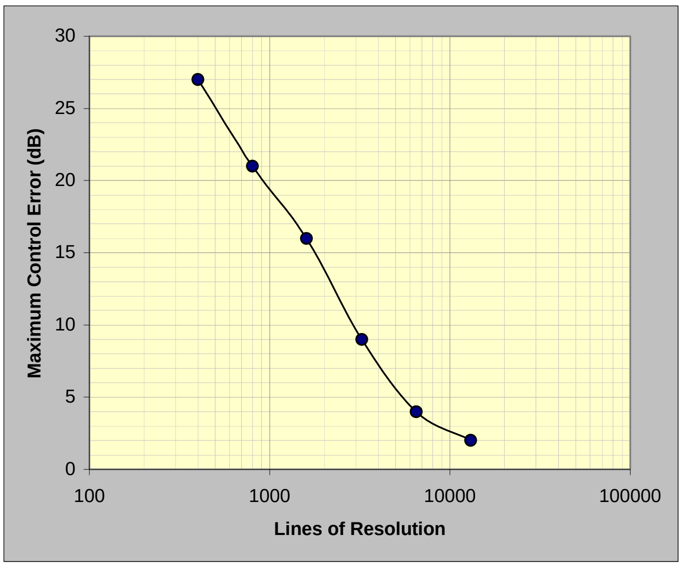


Figure 23: Maximum loop-error versus random control resolution shows that at least 8,000 lines of resolution are required to hold the challenge filter in control within ±3 dB over the NAVMAT band.

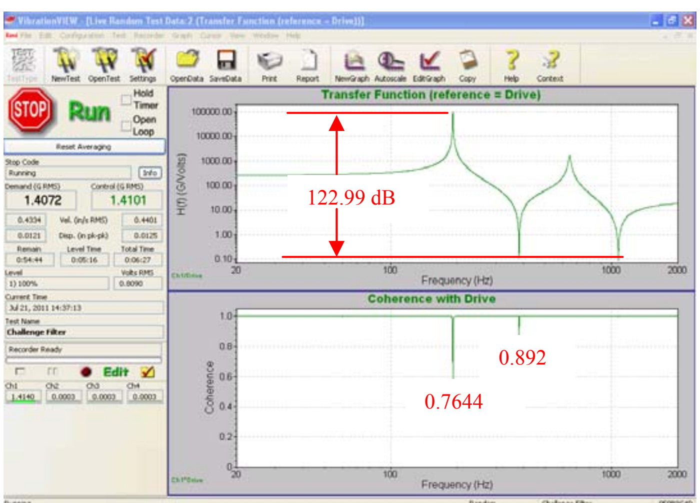


Figure 24: 13,000 line Transfer Function of the challenge filter shows over 120 dB of dynamic range on the VR9500; the corresponding Coherence Function indicates a clean measurement.

Sine Algorithm and Output Test Using a "high-Q" Filter to Simulate a Shaker and Structure

The challenge filter may also be used in the setup in Figure 18 to measure dynamic range with a swept-sine test. For this test a flat *Demand* amplitude is set, and the sine frequency is swept through the

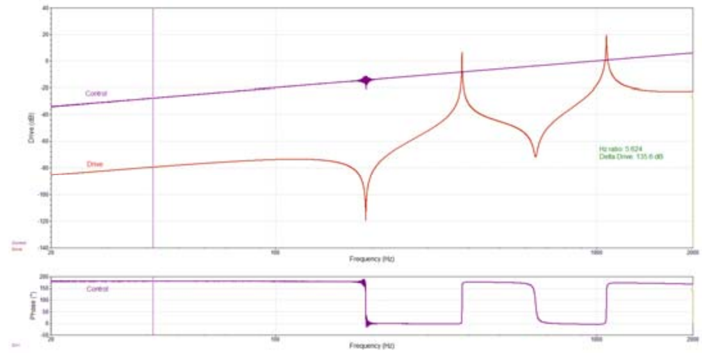


Figure 25: VR9500 Control and Drive signals during a swept-sine test; dynamic range exceeds 135 dB. The test profile has a change in demand amplitude (0.02 V to 2 V) because the dynamic range of VR9500 exceeded the dynamic range of challenge filter.

peaks and notches of the challenge filter. The implication of this is only the dynamic range of the control loop and the controller output signal are tested. Since the controller input is specified to maintain one amplitude level for the duration of the test, the dynamic range of the controller input remains untested. This test measures the relative values of the largest achievable output, which is required at the lowest notch in the filter transfer function, to the smallest achievable output (i.e. just above the noise floor) at the frequency of the highest peak in the filter transfer function.

Again, recall that to measure the dynamic range the test signals must be carefully crafted such that the highest output corresponds to the maximum range of the system. To successfully run this test, the loss at the lowest notch must be known in advance. For the sample case here, the filter transfer function has a 50dB loss at the lowest notch, so the demand level must be at least 50dB lower than the maximum achievable output voltage. For a controller with a 10 V maximum voltage we choose a demand level of 0.01 V_{PEAK}, or 60dB below the maximum.

Figure 25 illustrates a swept-sine test using a constant-amplitude 0.01 V_{PEAK} *Demand*. Note clearly evident *Drive* signal swings of 135 dB.

Conclusions

Eight tests of various aspects of controller dynamic range have been demonstrated and discussed. The two-tone input and single-tone output tests, while useful for measuring harmonic distortion, were demonstrated to be poor measures of dynamic range. Their results depend more on the length of the FFT used for analysis than on the actual dynamic range of the device. The effective bits test was also presented. This test, while difficult to perform, does give a good measure of the signal-to-noise ratio of the system.

The use of closed-loop random control challenge filter which simulates a lightly-damped system was also discussed. This test was shown to be prone to incorrect evaluation unless the results are verified independent from the controller's display. It also provides an inaccurate, and inflated, measure of the dynamic range of the system. This test is useful for exercising the control loop in a realistic manner, and is a good test of the line resolution of the controller. However, the dynamic range numbers given by this test can be more than 10 dB greater than the true dynamic range of the system, so this test should not be used as a measure of dynamic range.

Three loop-back tests were discussed, which test the controller inputs and outputs simultaneously, without requiring any external equipment. These three tests are simple to perform, and combined give a good insight into the true capabilities of the controller.

At heart, the limits of a controller are determined by the maximum signal level, the noise floor, and the harmonic distortion characteristics of the system. The various measures of dynamic range will depend on these characteristics, but as was shown in this paper, dynamic range measurements can vary widely depending on how it is measured, and who is doing the measurement. Because of this, comparisons of controllers using dynamic range numbers are difficult at best. For unbiased comparisons, it is better to compare the maximum signal level, the noise floor, and the harmonic distortion characteristics of the systems.

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