Random Vibration Kurtosis Control

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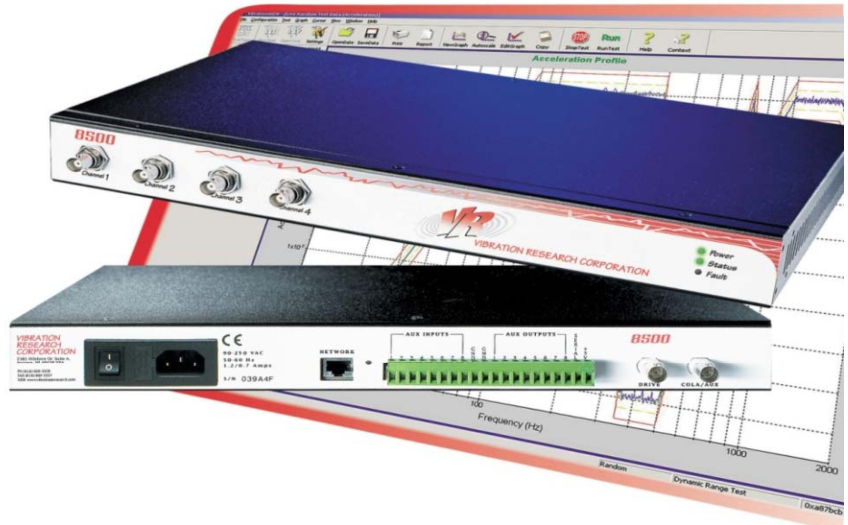
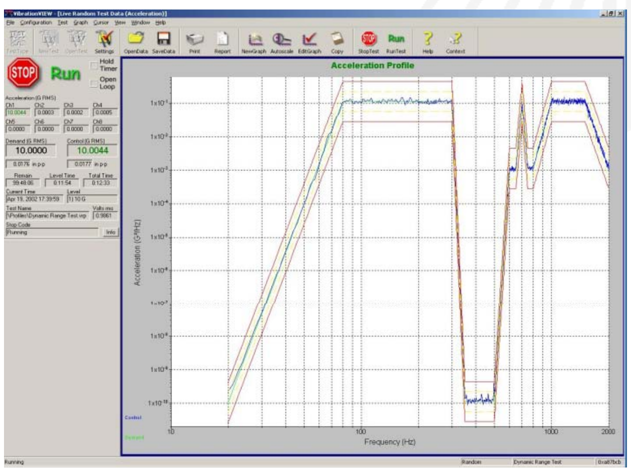




Presentation Summary

- What is kurtosis?
- Why are we interested in kurtosis?
- Kurtosis in the resonance?
- Papoulis Rule / Central Limit Theorem.
- Test-Shaker with Resonating bar.
- Control Random ED shaker kurtosis?
- How does this relate to the real world?

The Problem

Definition of the Problem

- Traditional random testing does not always find failures that occur during the life of a product.
- This is likely because the product experiences high G forces in actual use that are higher than traditional random testing generates

Kurtosis

- Show of hands: Who has heard of the term “Kurtosis” before today?

- Definition in terms of statistical moments
  - Mean is the 1st moment
  - Variance is the 2nd moment
  - Skewness is normalized 3rd central moment
  - Kurtosis is normalized 4th central moment

Calculating Kurtosis

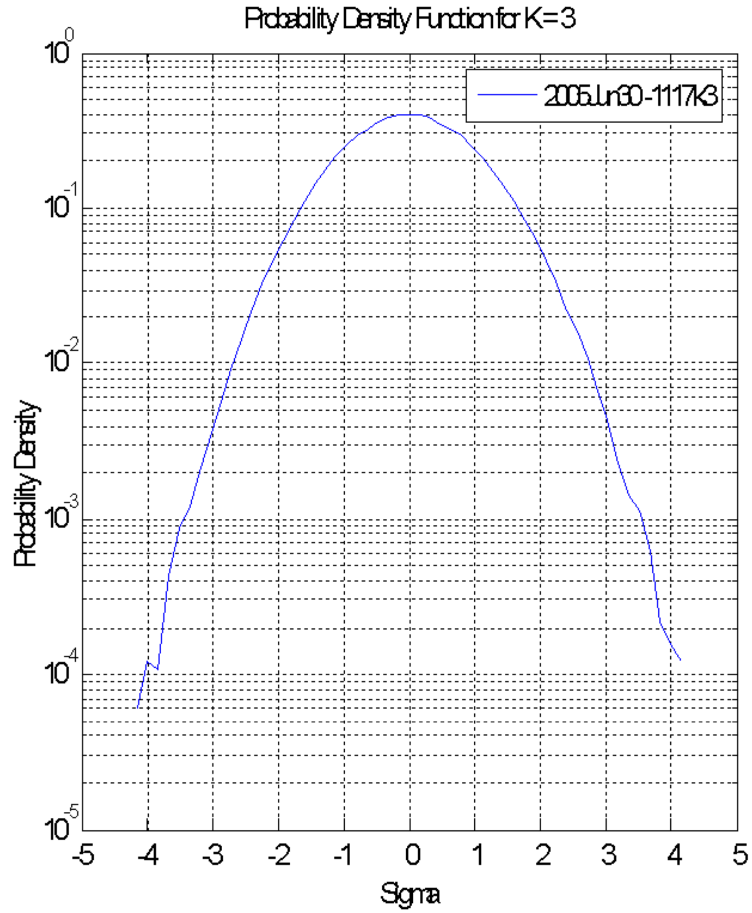
- The basic function for calculating kurtosis for zero-mean data is:

$$\frac{\text{average}(\text{data}^4)}{\text{average}(\text{data}^2)^2}$$

- Different people normalize this value in different ways
  - As commonly used, Gaussian kurtosis = 3
  - Microsoft Excel subtracts 3, so Gaussian kurtosis = 0
  - Others divide by 3, so Gaussian kurtosis = 1

Traditional Random Testing

- Current random testing seeks to achieve a Gaussian distribution
  - "Normal" distribution
  - Concentrated around mean
  - Low probability of extreme values
  - Kurtosis = 3

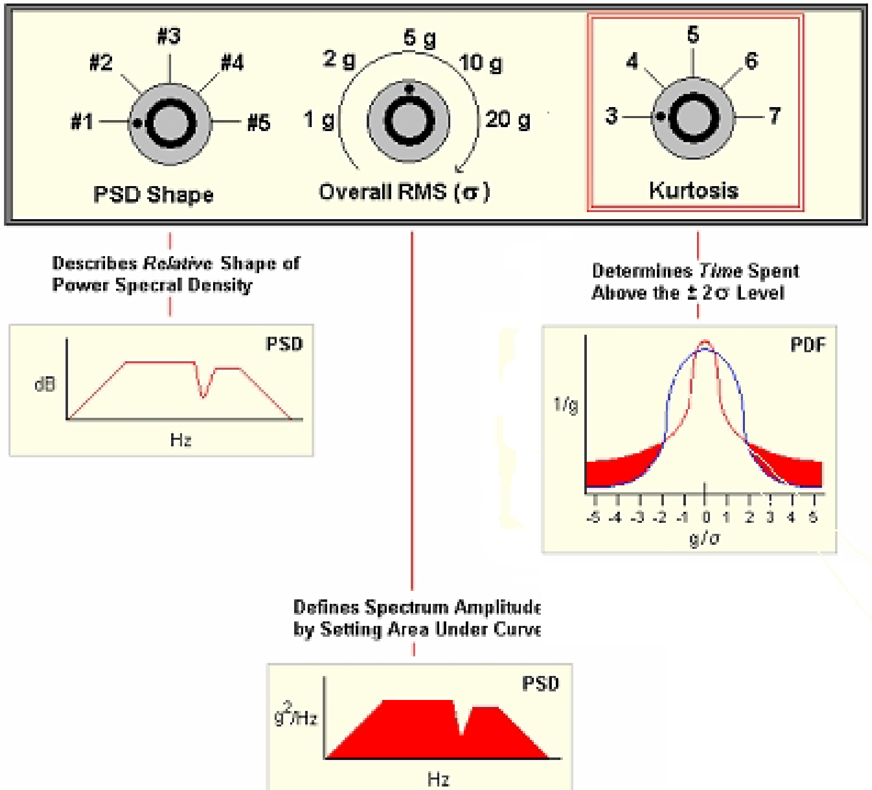


What is Missing on ED Shaker Controllers?

- Random vibration controllers have 2 basic “knobs”:
  - Frequency content - Power Spectral Density (PSD)
  - Amplitude level - RMS

- Need a third ‘knob’ to adjust the Kurtosis
  - Allows adjustment of the PDF (probability density function)
  - Increasing kurtosis = increasing peak levels
  - Allows the damage-producing potential of the test to be adjusted independent of the other two controls.

The Missing Knob



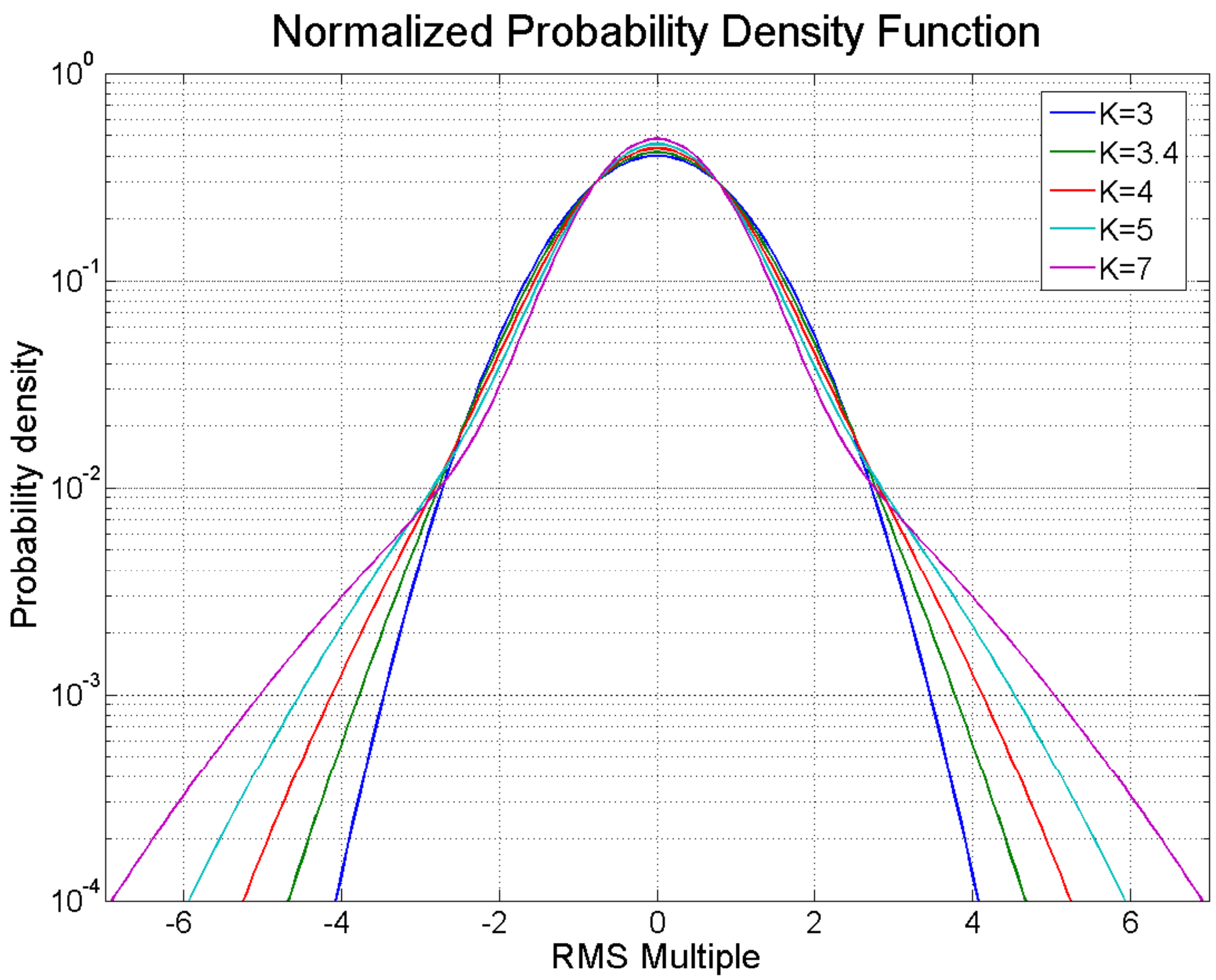
Kurtosis Control

- Objective is to control the amplitude distribution to achieve the higher peaks seen in field data
  - Spectrum is a measure of the frequency content
  - RMS is a measure of the amplitude
  - Kurtosis is a measure of the "peakiness"

- Solution is use a non-Gaussian vibration and control the Kurtosis

- This is what we call Kurtosion™
  - Method to simultaneously control Spectrum, RMS, *and Kurtosis*
  - Patent-pending

PDF Varies with Kurtosis



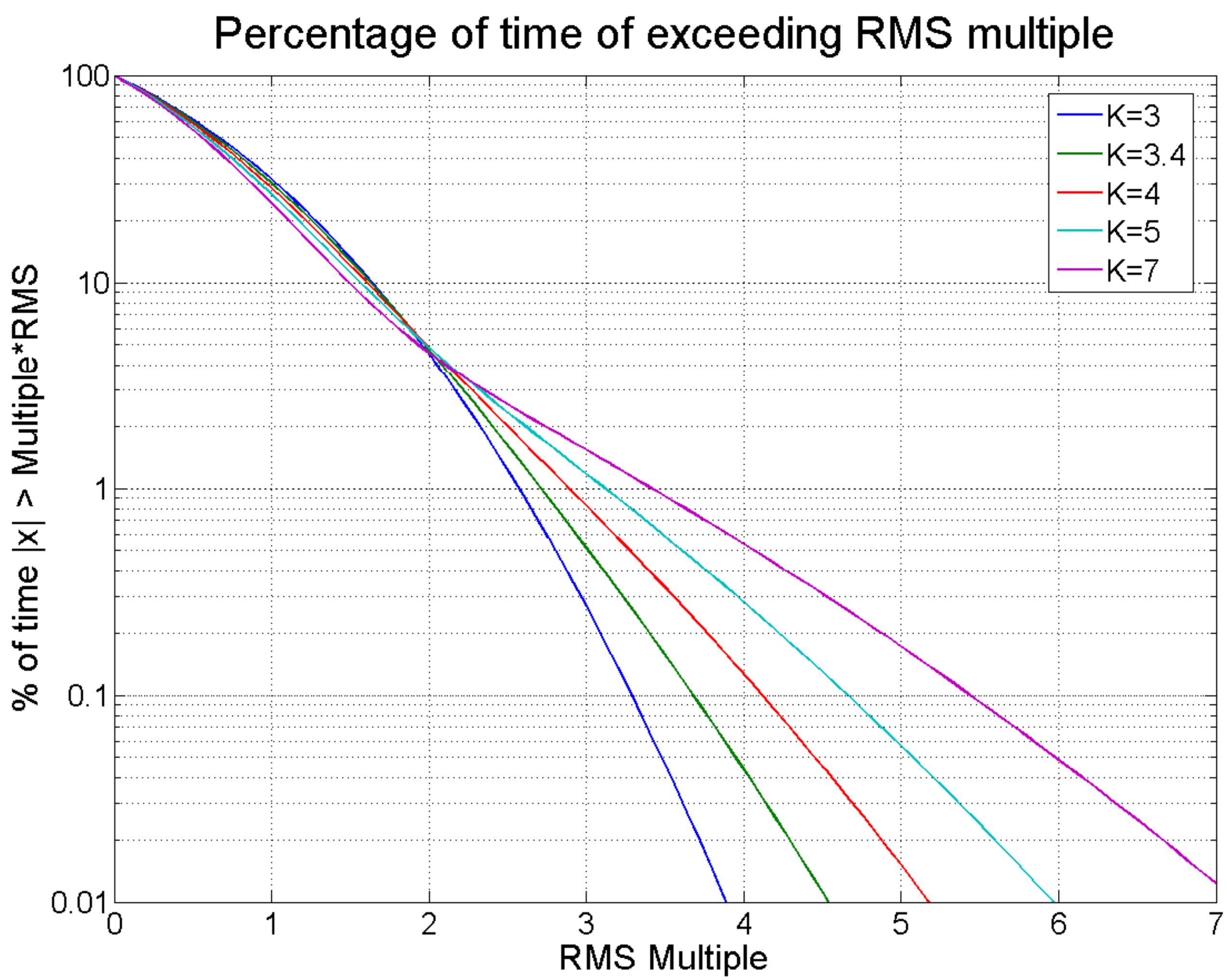
Increased Kurtosis = More Time at Peaks

Kurtosis = 3 is > 3σ
0.27% of time

Kurtosis = 4 is > 3σ
0.83% of time

Kurtosis = 7 is > 3σ
1.5% of time

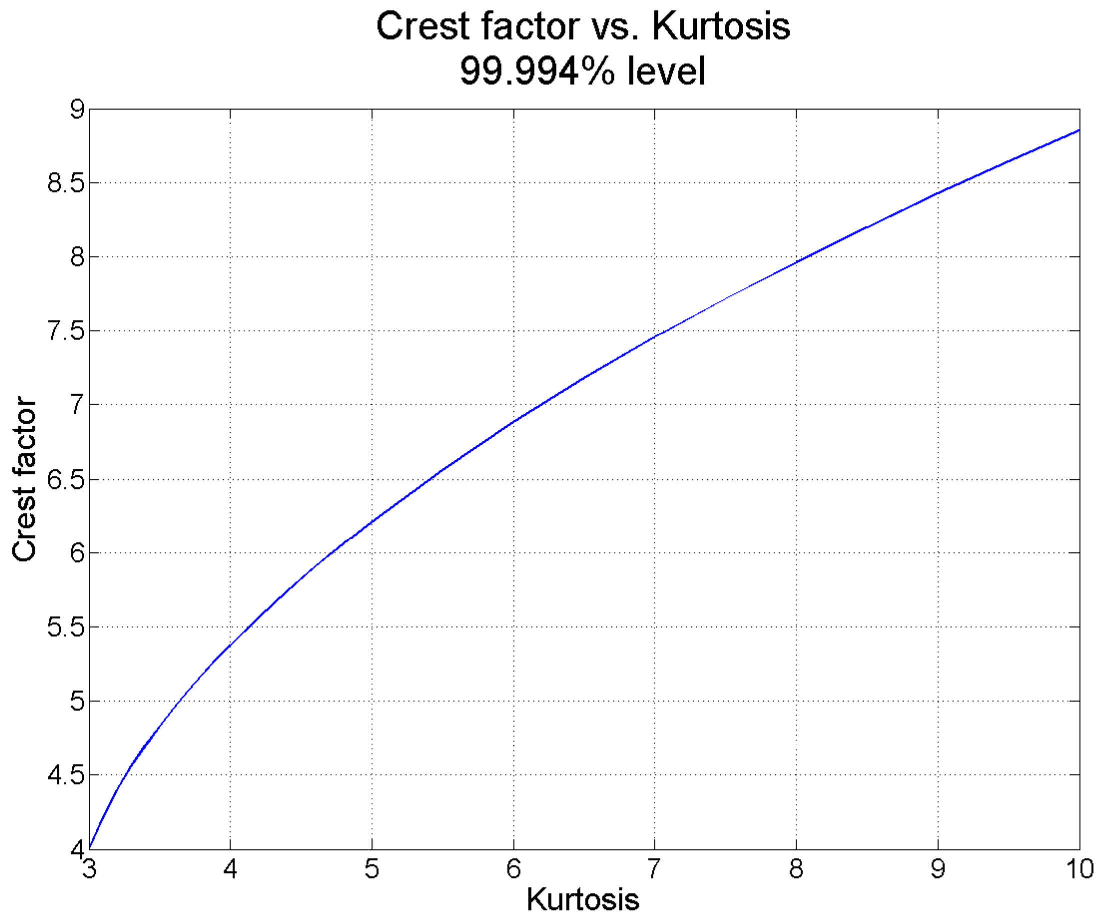
Note: 1.5% of a 1 hour test is nearly a full minute above 3σ



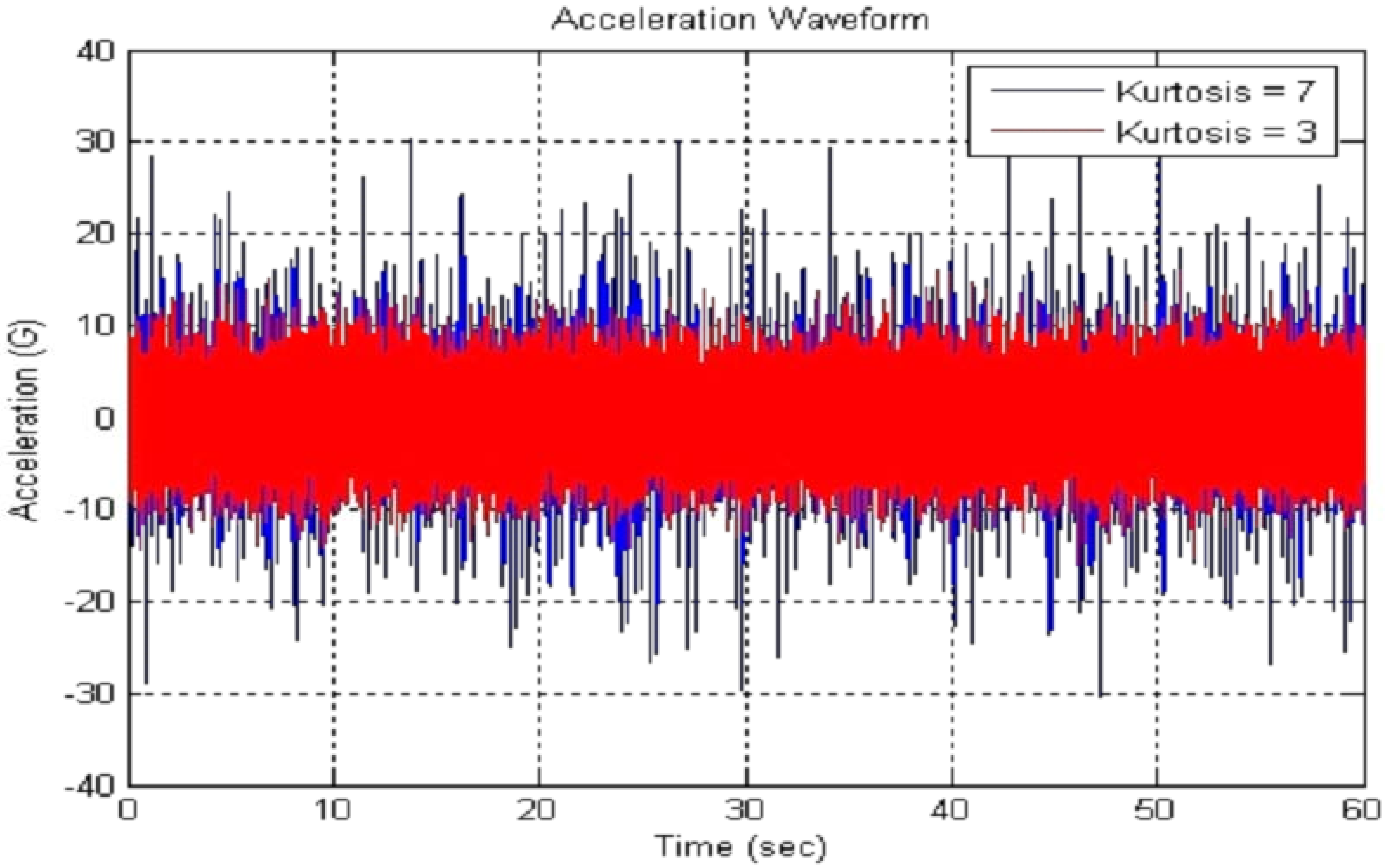
Increased Kurtosis = Higher Peaks

Crest Factor:
The ratio of peak to rms

Note:
Crest factor of the 99.994% probability level is plotted, as this is the 4 times rms (4 sigma) level for a kurtosis=3 random test. Also, this is the typical maximum peak seen on a kurtosis=3 random test.



Waveform Comparison



Useful Properties for Kurtosis Control

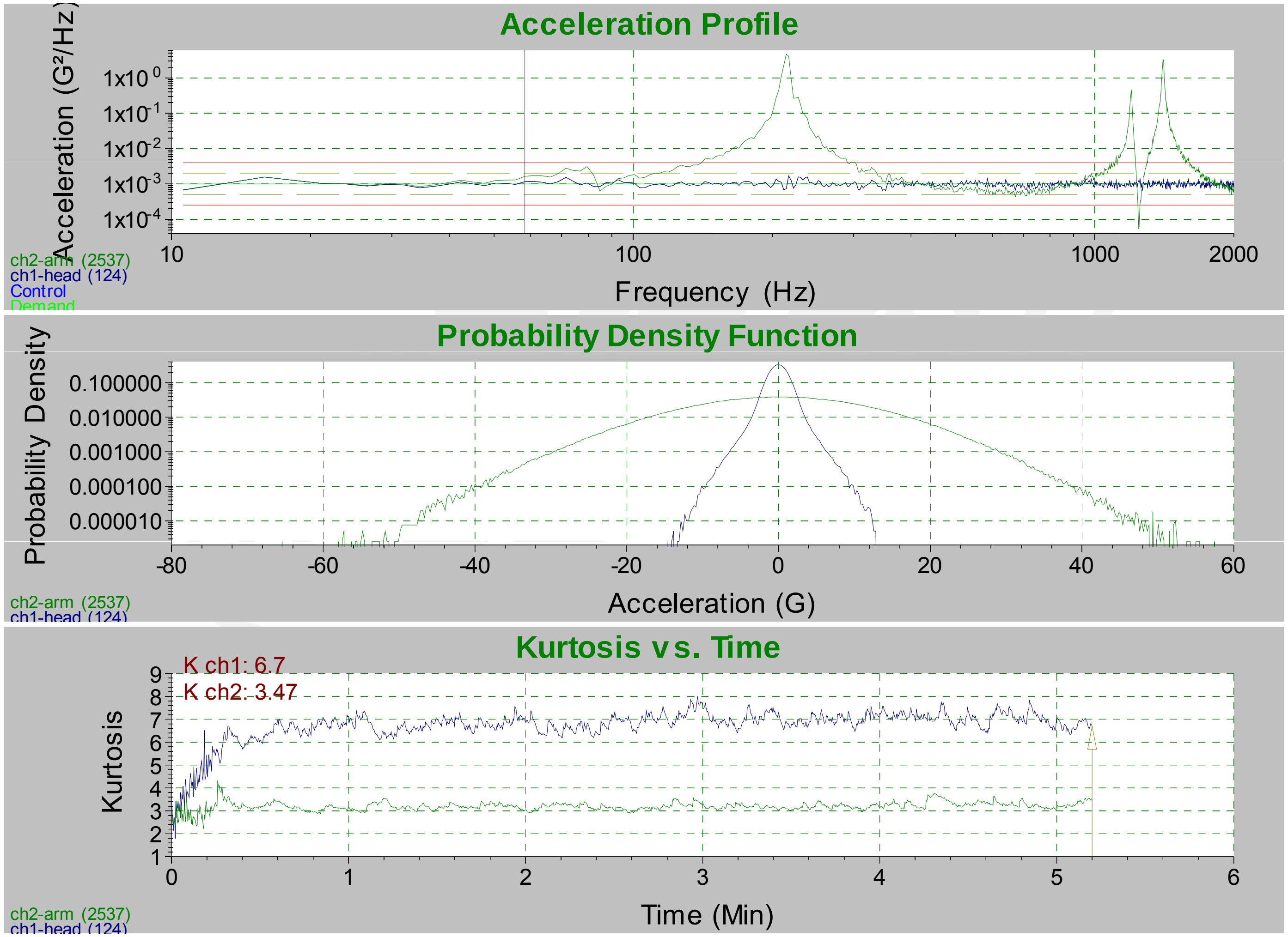
- ♦ Set kurtosis independent of RMS.
- ♦ Set kurtosis without affecting PSD.
- ♦ Increase kurtosis over the full spectrum.
- ♦ Dynamic range of the controller is maintained.
- ♦ Apply kurtosis even in a resonance.
- ♦ Note Papoulis Rule requirements.

Papoulis Rule

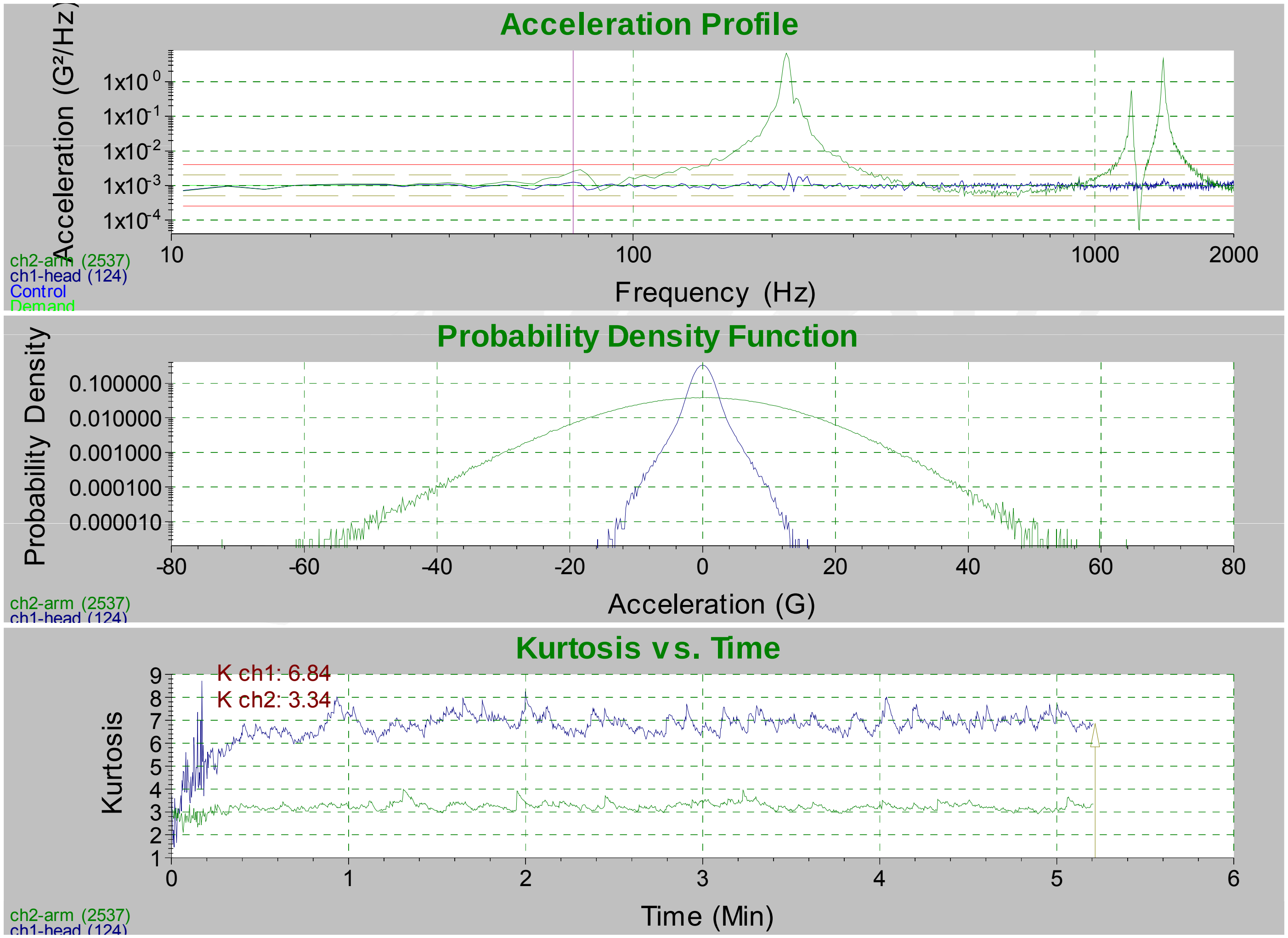
- Papoulis' Rule states that filtered waveform *tends towards* Gaussian
- Bound is proportional to the 14th root of the filter bandwidth.
  This is an extremely weak limit.
- Bound is constant across all values of the CDF.
  Weak limit on the tails of the distribution which give Kurtosis
- Practical results of Papoulis' Rule
  - Kurtosis at the resonance gets reduced from the kurtosis of the excitation signal
  - For practical Q factors, there will still be some significant kurtosis at the resonance.

Resonating Bar Test Setup

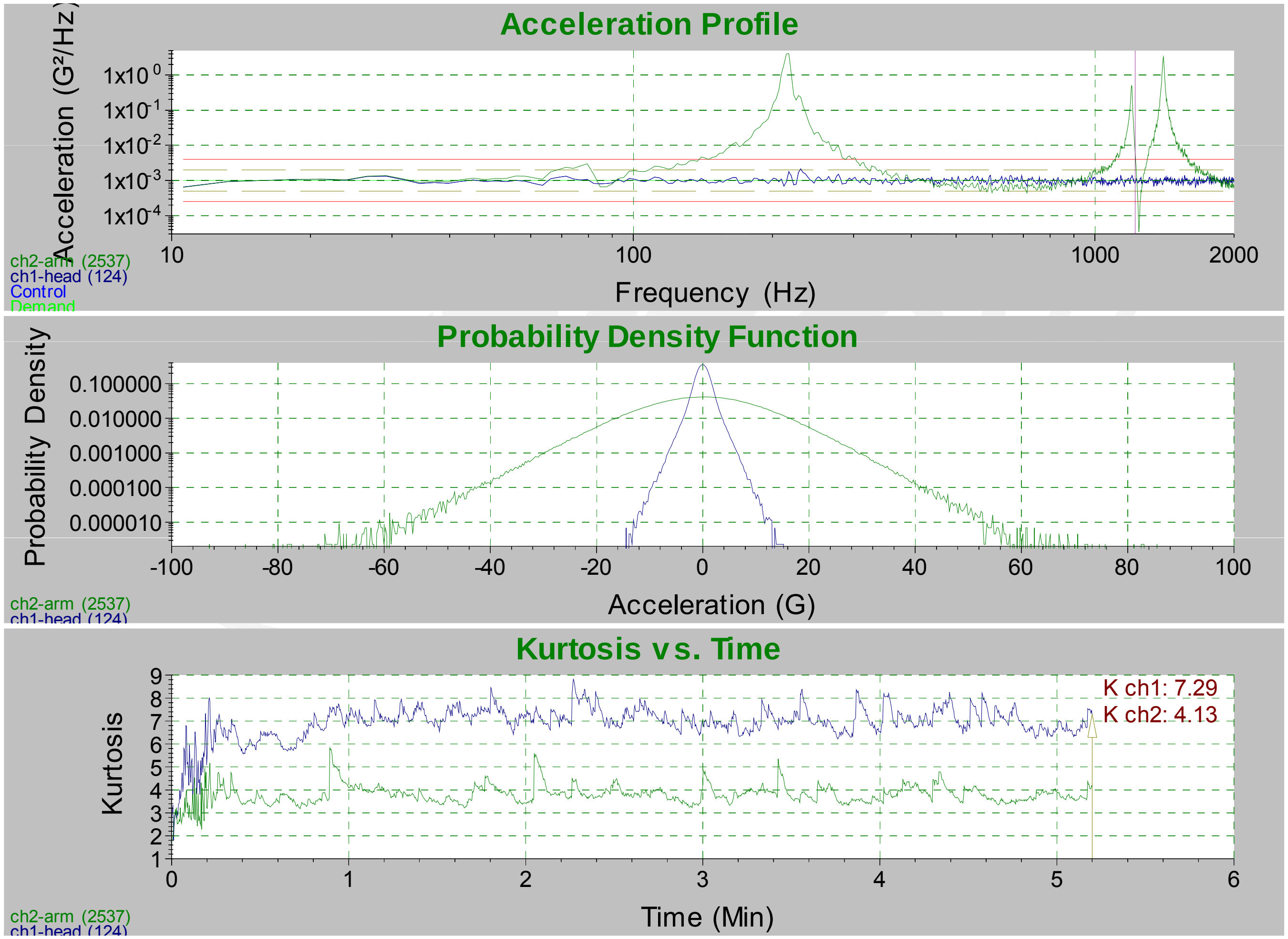
10,000 Hz Transition Frequency



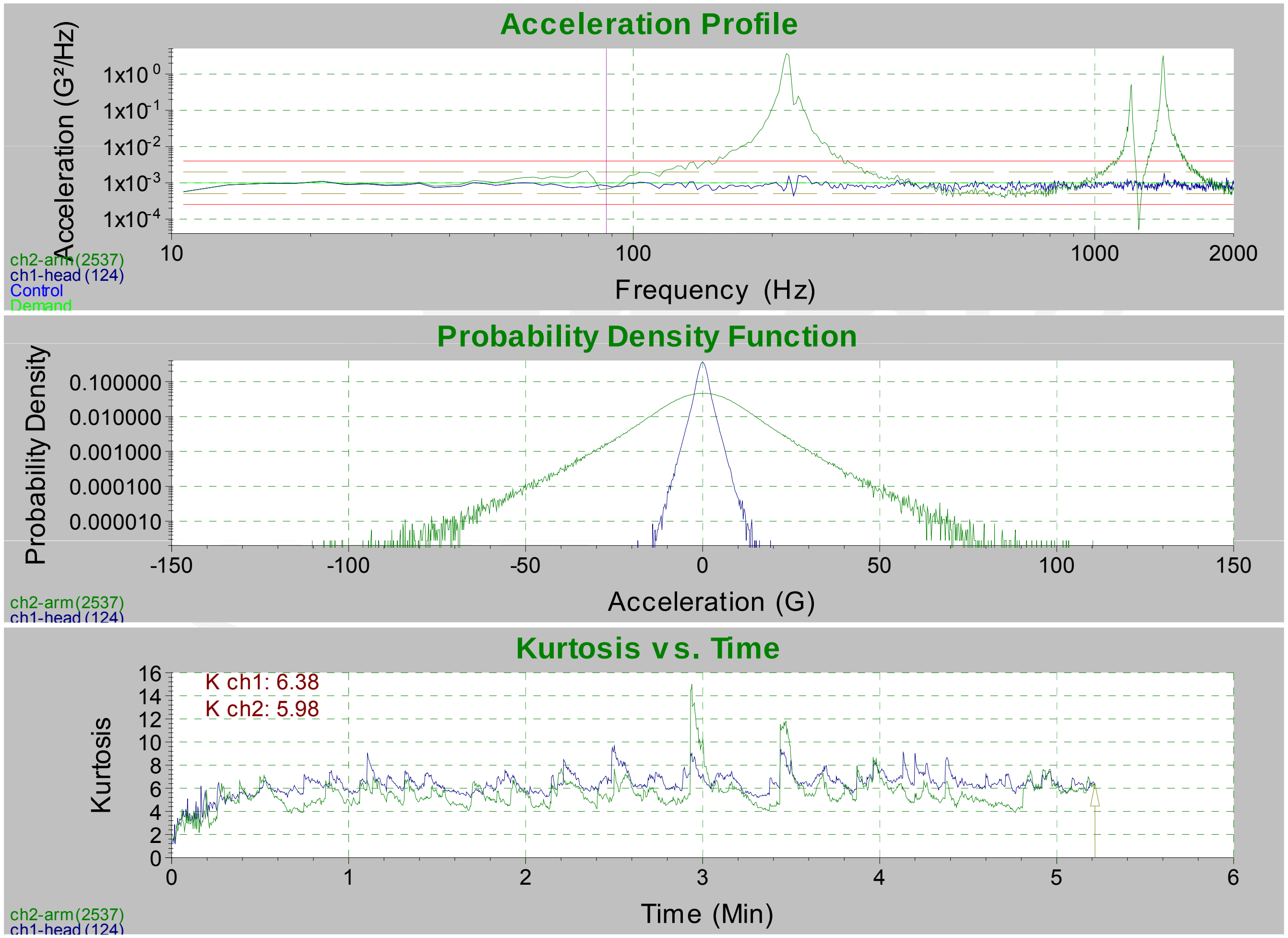
1,000 Hz Transition Frequency



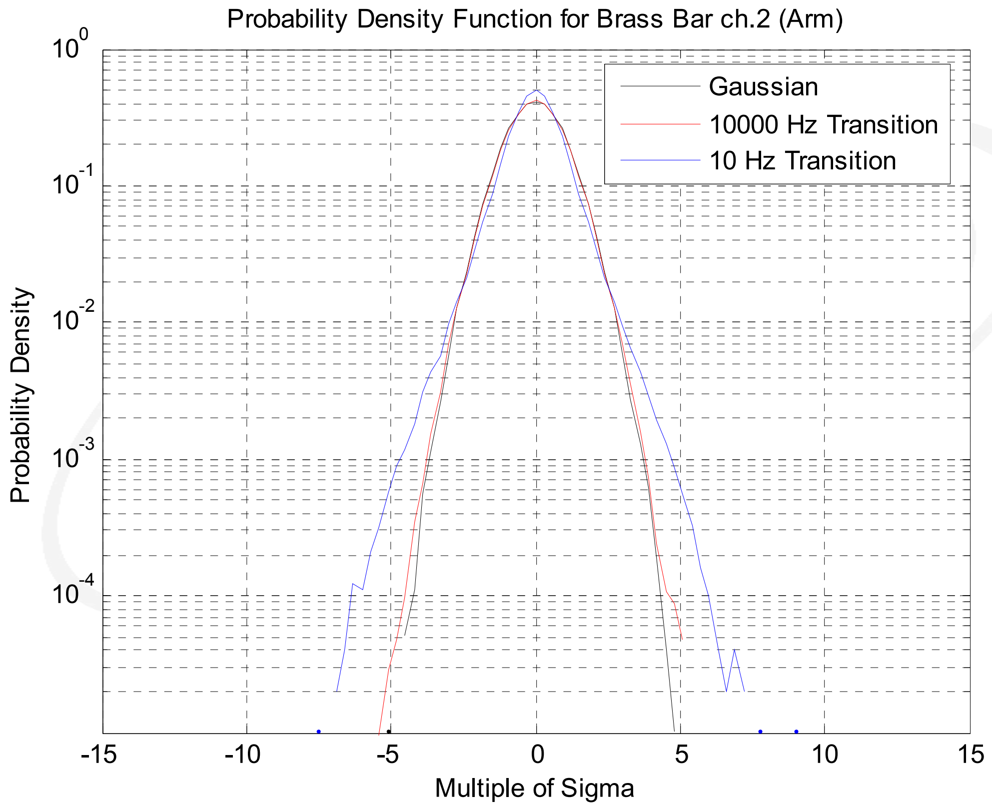
100 Hz transition Frequency



10 Hz Transition Frequency

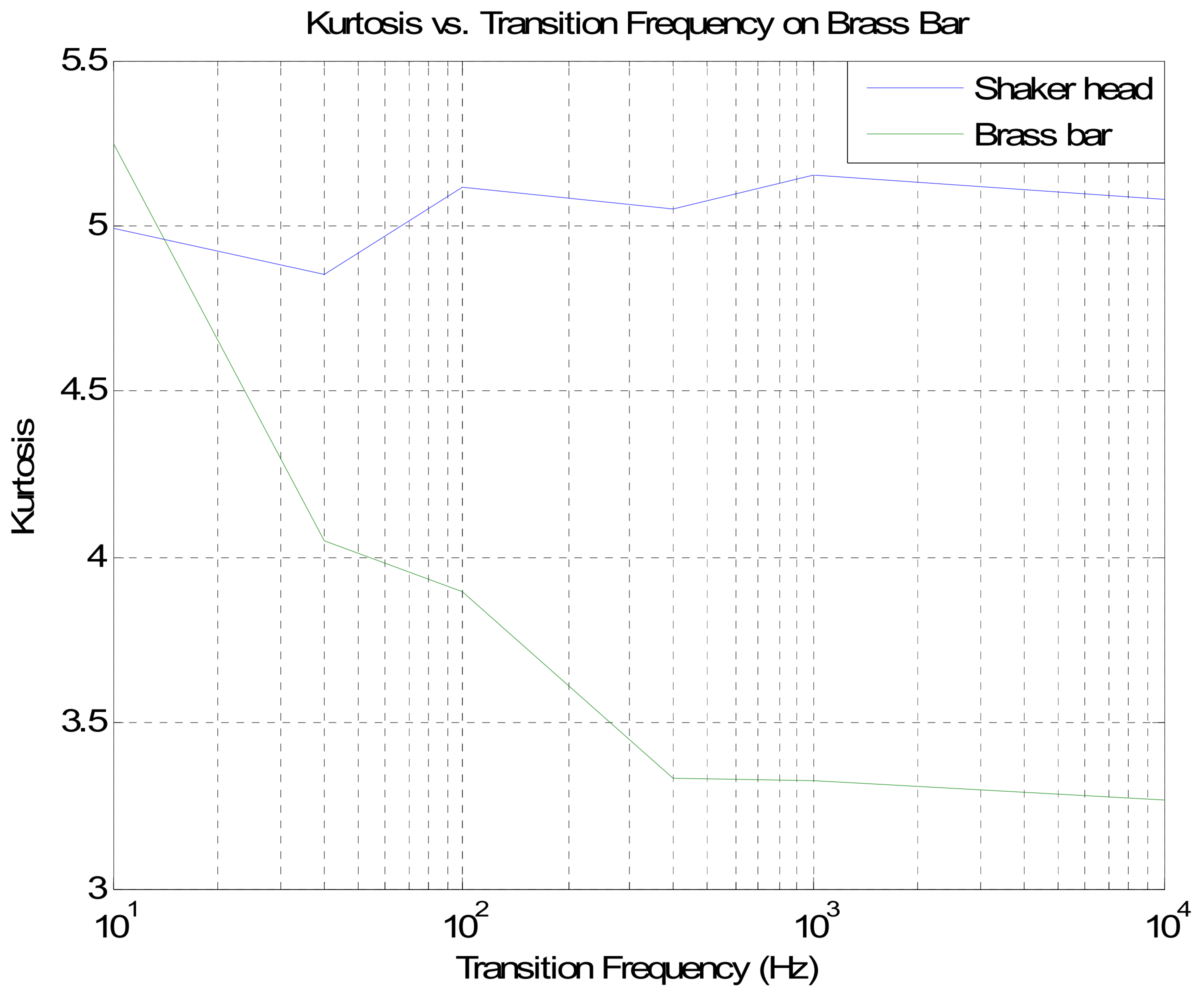


Effect of Transition Frequency on PDF



PDF for the Brass Bar (Arm) Data for tests with Gaussian distribution, Kurtosis = 5 (Transition 10,000 Hz) and Kurtosis = 5 (Transition 10 Hz).

Effect of Transition Frequency on Kurtosis



Control on Resonant Beam

OBJECT		*TRANSITION FREQUENCY*	*KURTOSIS SETTING*	*KURTOSIS HEAD*	*KURTOSIS ARM*
Brass Bar	10-2000 Hz	10000 Hz	5	10.5	3.83
Brass Bar	10-2000 Hz	10000 Hz	5	11.0	3.38
Brass Bar	10-2000 Hz	10000 Hz	5	10.3	3.85
Brass Bar	10-2000 Hz	10 Hz	5	4.99	5.63
Brass Bar	10-2000 Hz	10 Hz	5	4.80	4.69
Brass Bar	10-2000 Hz	10 Hz	5	5.78	5.0

Results from Brass Bar Vibrations where the end of the bar was controlled and the head responded.

Lower Transition Frequency allows controller to easily produce the desired kurtosis value at resonance.

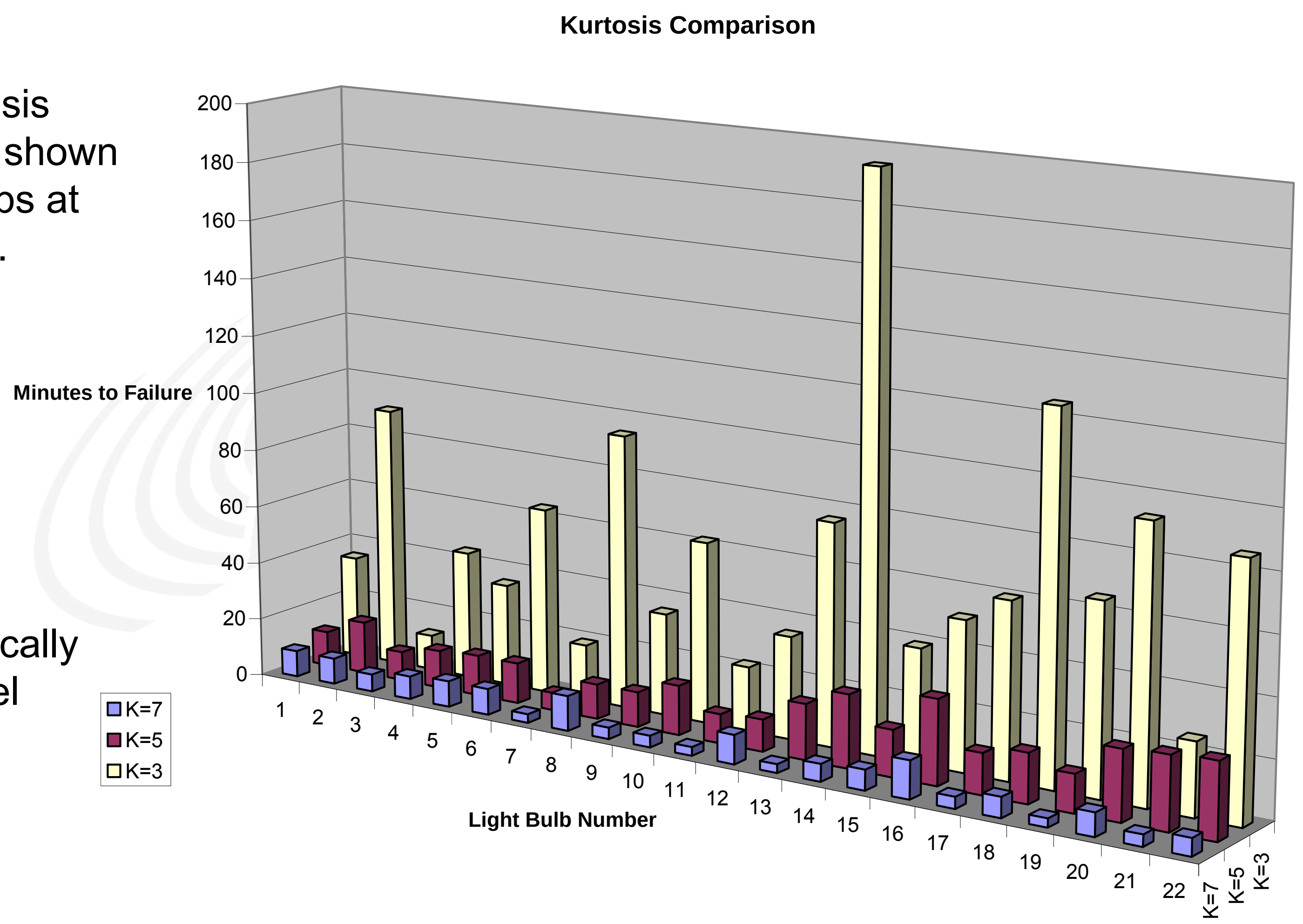
Light Bulb Test - revisited

- Previous papers examined the effect of increasing kurtosis on failure of light bulbs.
- As kurtosis was increased, failure time decreased.
- Now, we run a test with constant kurtosis, and vary the transition frequency

Previous Test Kurtosis Results

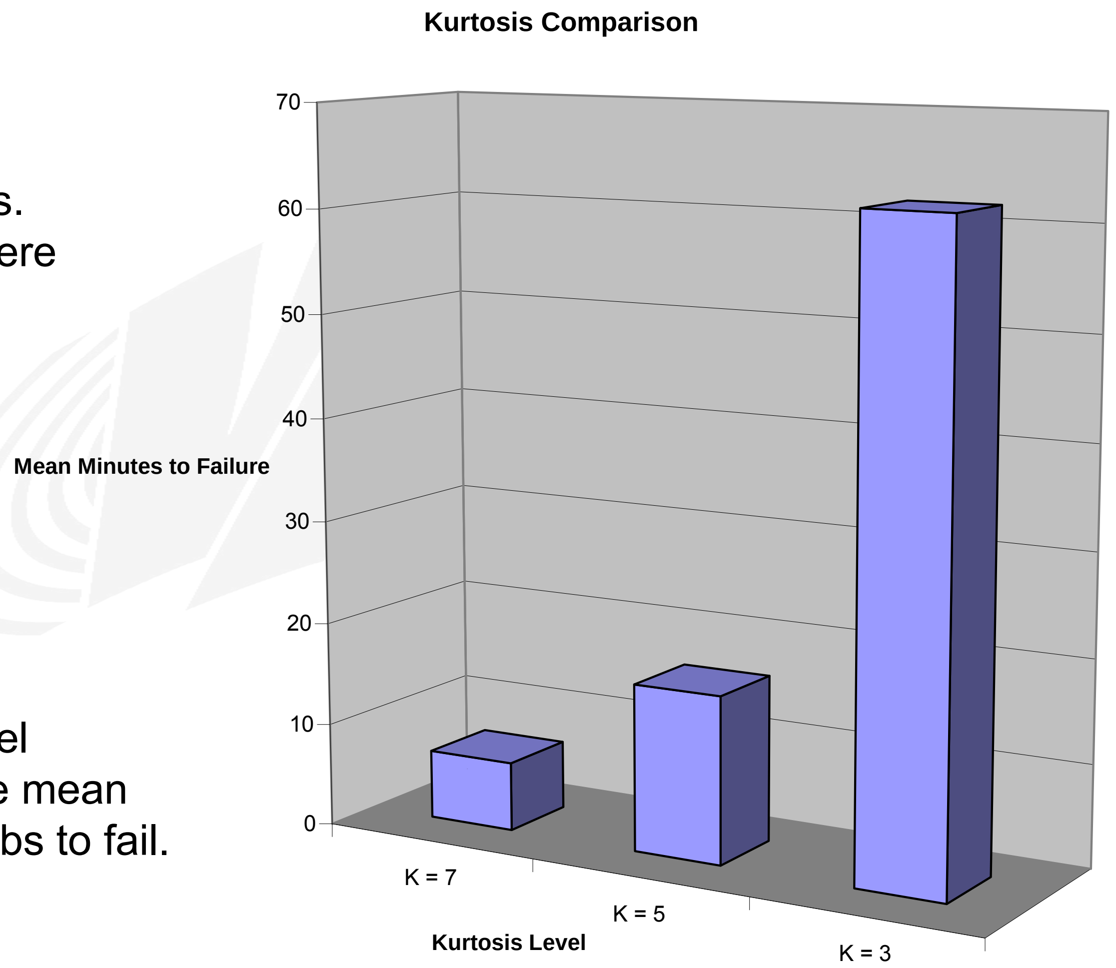
Consider the Kurtosis Comparison Chart shown here for the 22 bulbs at each kurtosis level.

The time it took to complete a test decreased dramatically as the kurtosis level increased.



Previous Test Kurtosis Results (cont)

Consider the graph of the Mean Minutes to Failure vs. Kurtosis Level as shown here



The increased kurtosis level dramatically decreased the mean time it took for the light bulbs to fail.

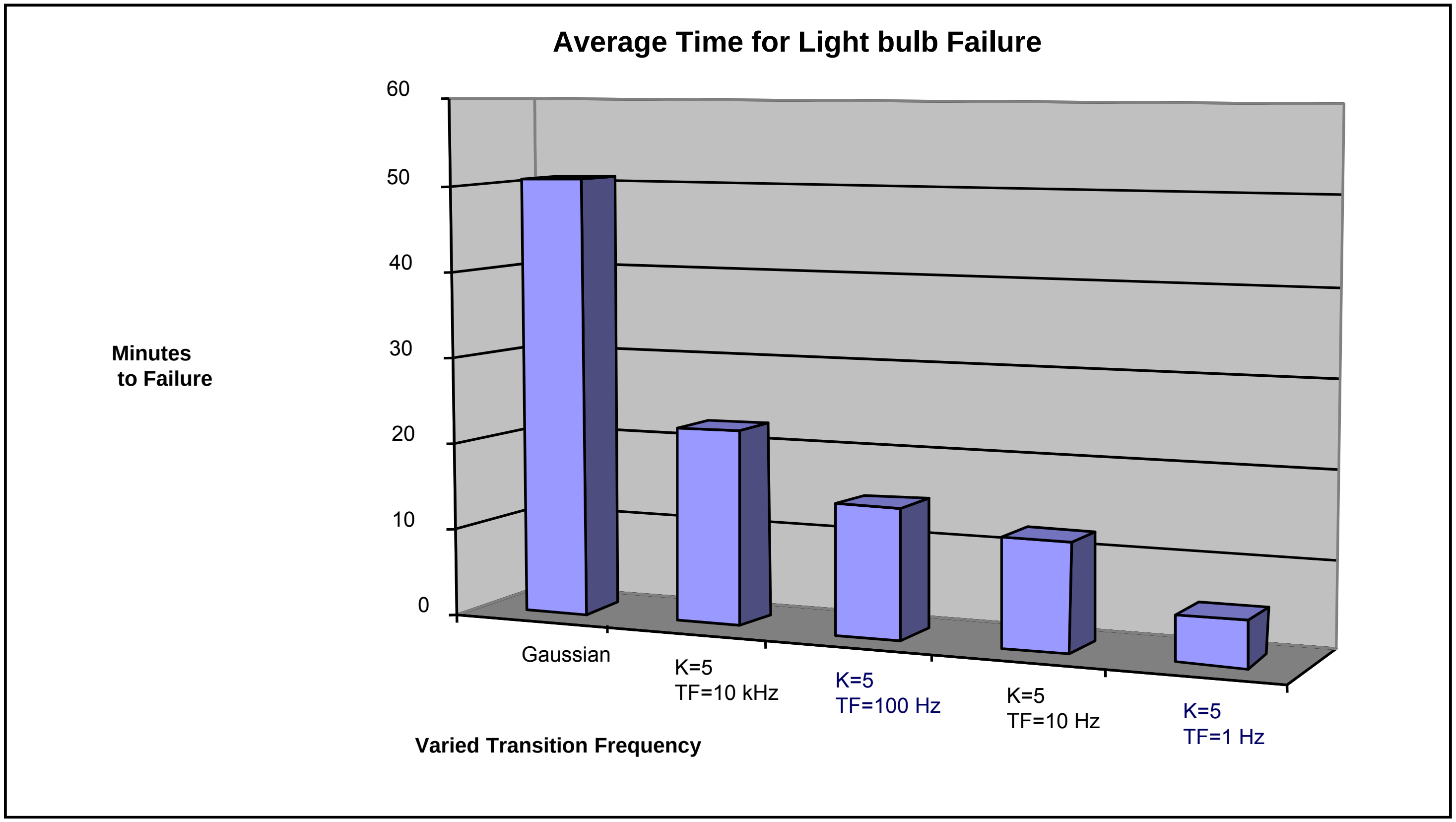
New Light Bulb Test

Minutes to Failure

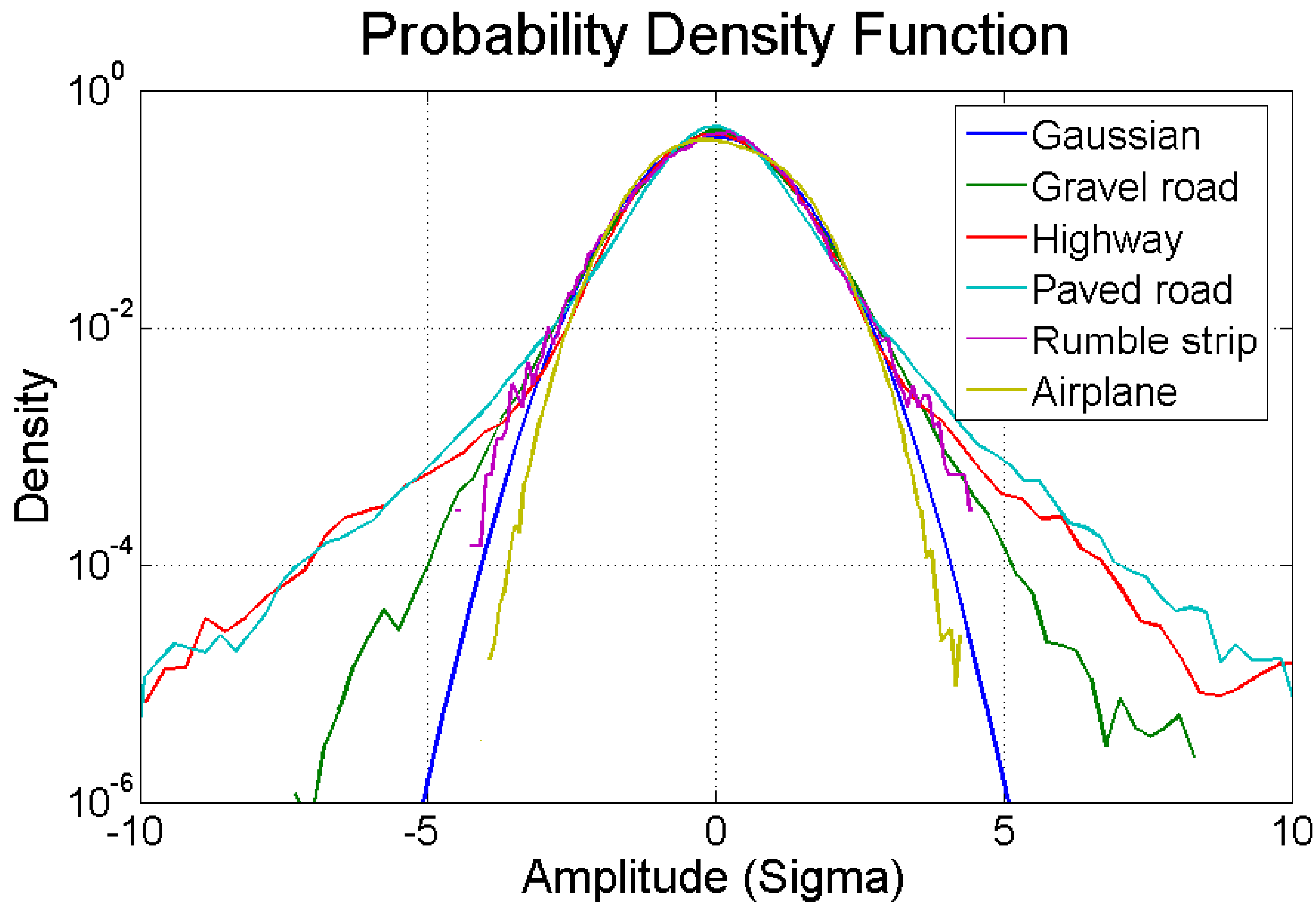
Gaussian 1.00 X RMS	Kurtosis 5 TF:10000 Hz	Kurtosis 5 TF: 100 Hz	Kurtosis 5 TF: 10 Hz	Kurtosis 5 TF = 1 Hz
32	7	12	6	2
62	25	12	15	6
93	30	13	20	8
102	37	21	24	10
41	11	10	5	2
41	13	15	12	2
45	20	17	13	4
52	27	19	14	11
18	19	11	8	2
36	21	14	8	3
44	28	15	9	4
45	32	20	14	10
50.9	**22.5**	**14.9**	**12.3**	**5.3**

Light bulb failure times at different Transition Frequency values. Note that as the Transition Frequency value decreases the time to failure also decreases.

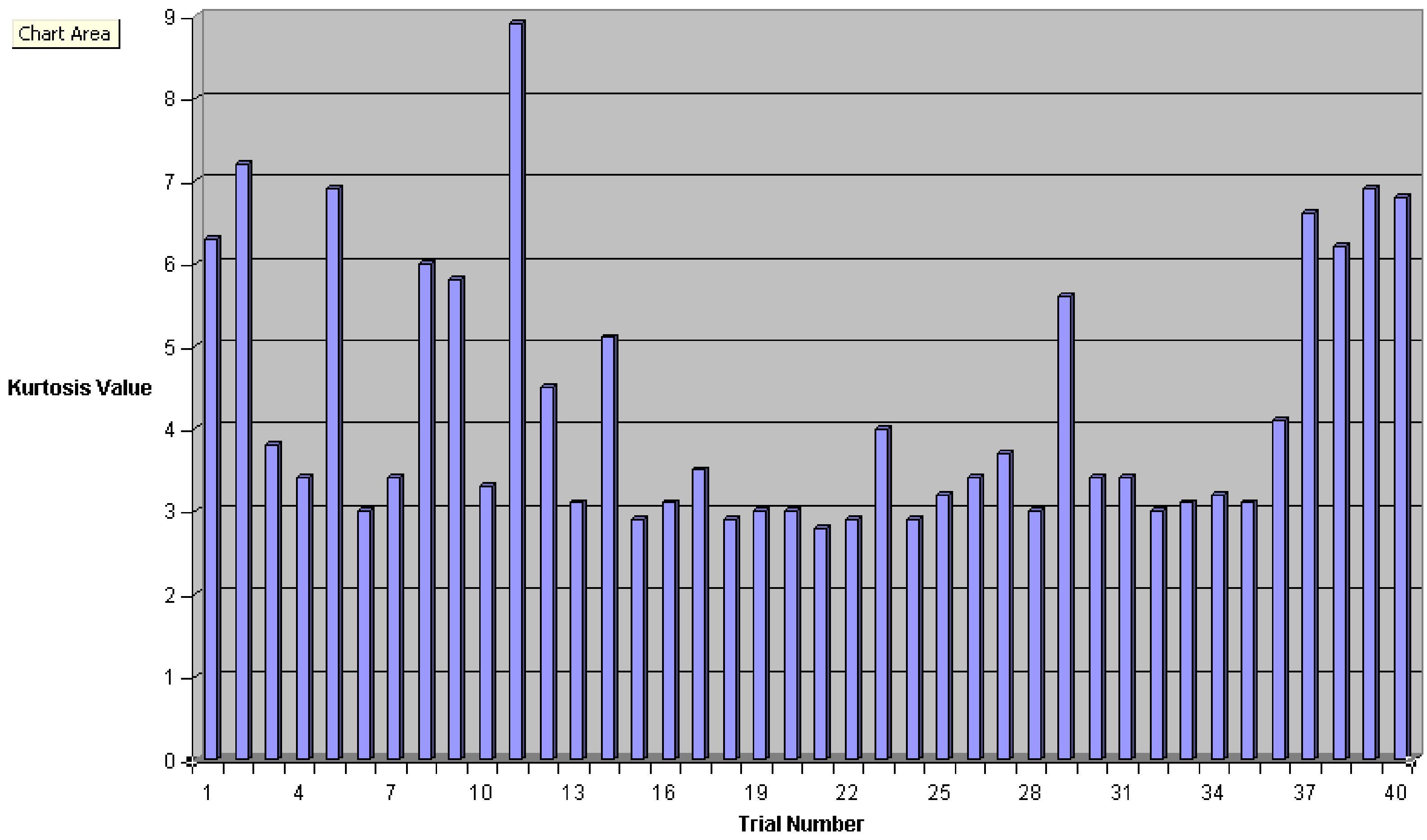
Minutes to Failure



What is the Kurtosis of the Real World?

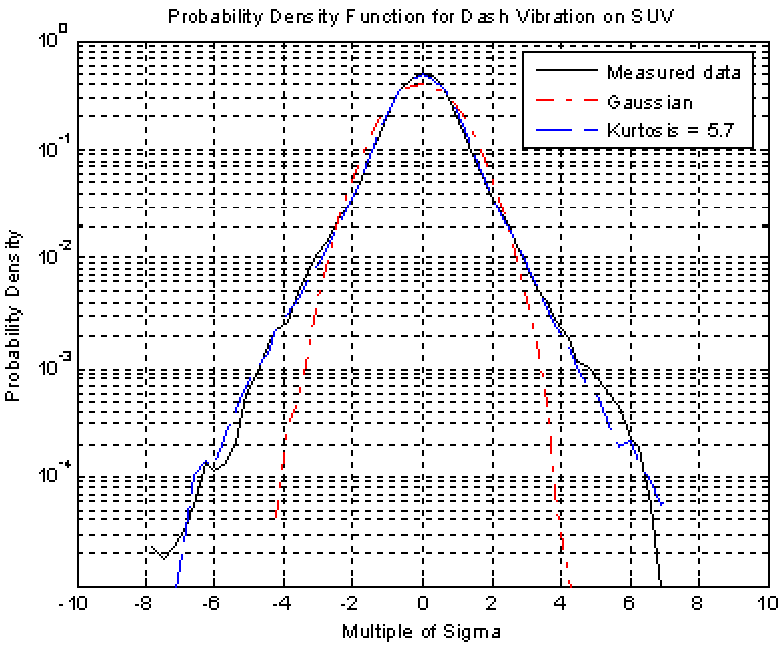


Kurtosis Values of All Tests



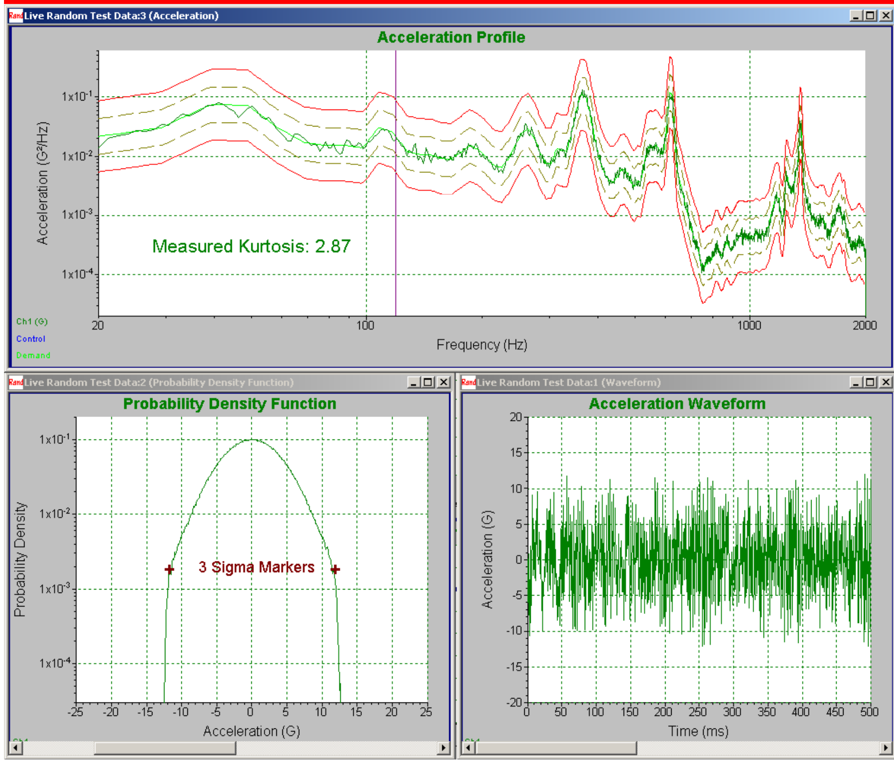
PDF of a Real Life Environment

Oldsmobile Bravada, dashboard vibration
As vehicle travels down I-196.

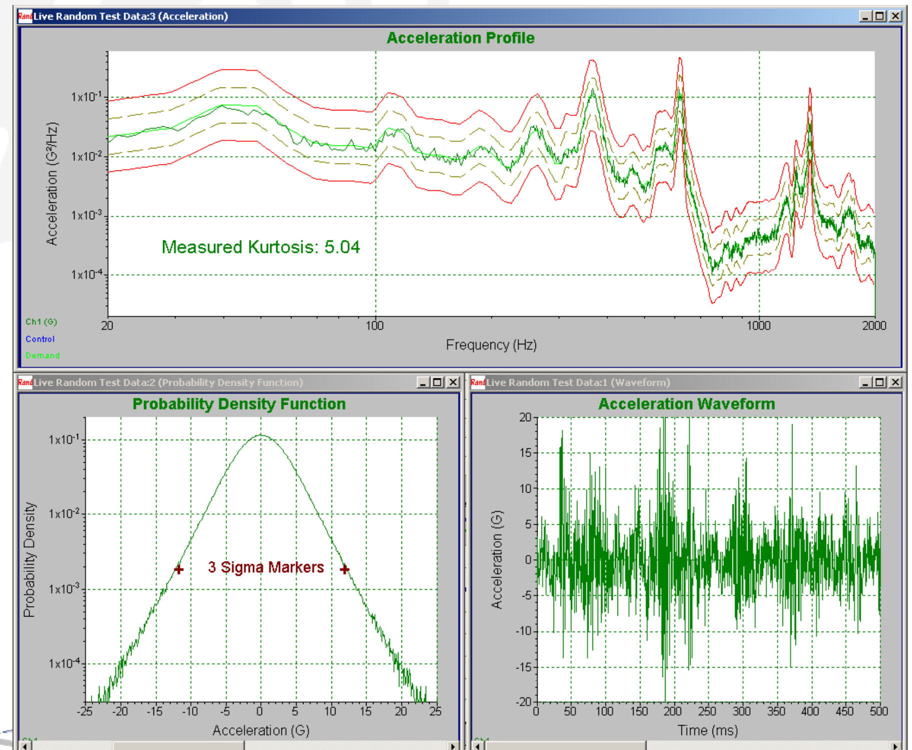


Probability Density Function for Gaussian, and actual, and controlled kurtosis

Random Test with PDF of a Real Life Environment



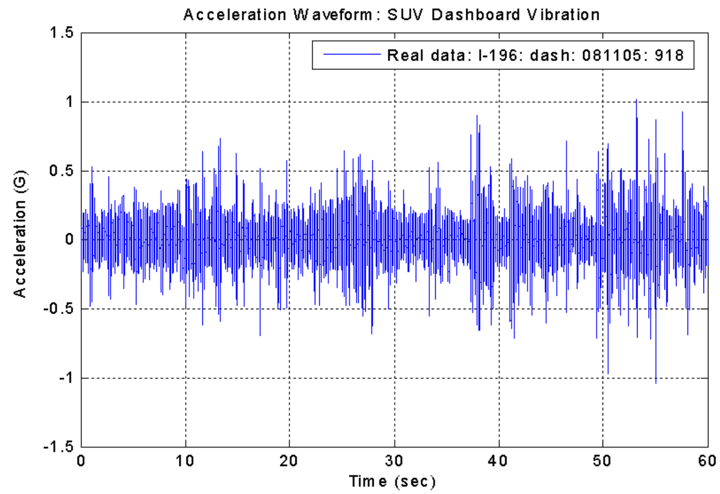
Spectrum defined by field measured data
Traditional random test with 3 sigma clipping



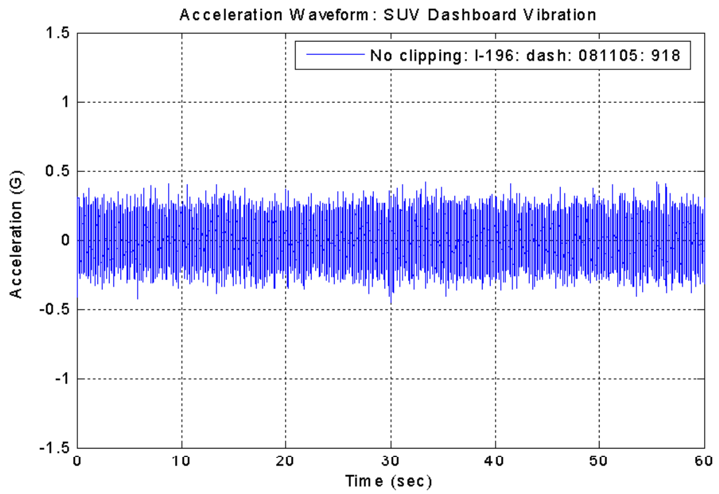
Same spectrum defined by field measured data
Now with Kurtosis Control set to 5

Time Waveform Illustrations

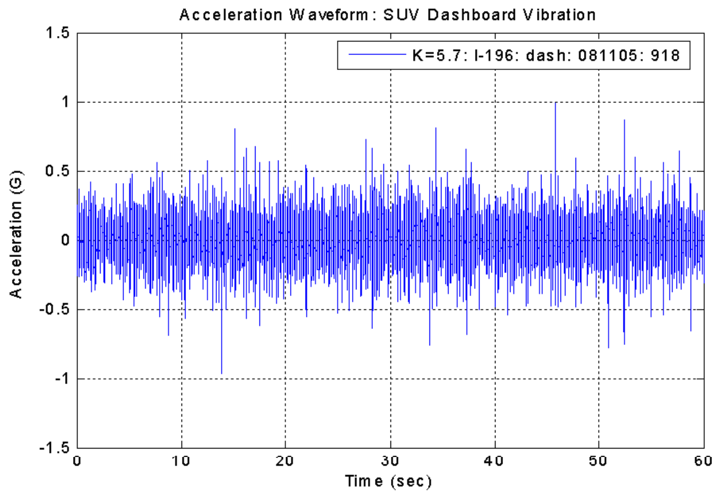
60 seconds of some actual field data K=5.7



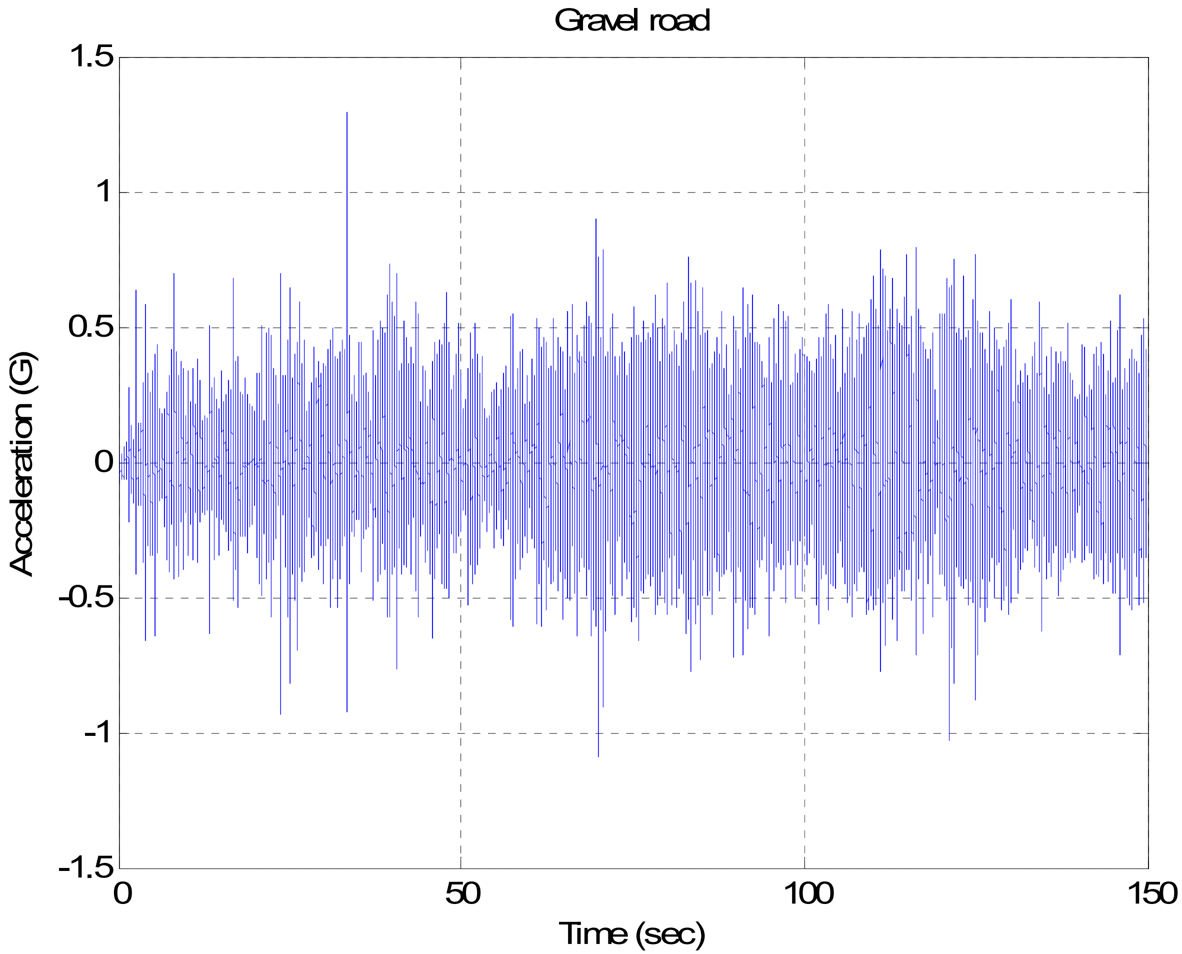
60 seconds of some Traditional random K=3



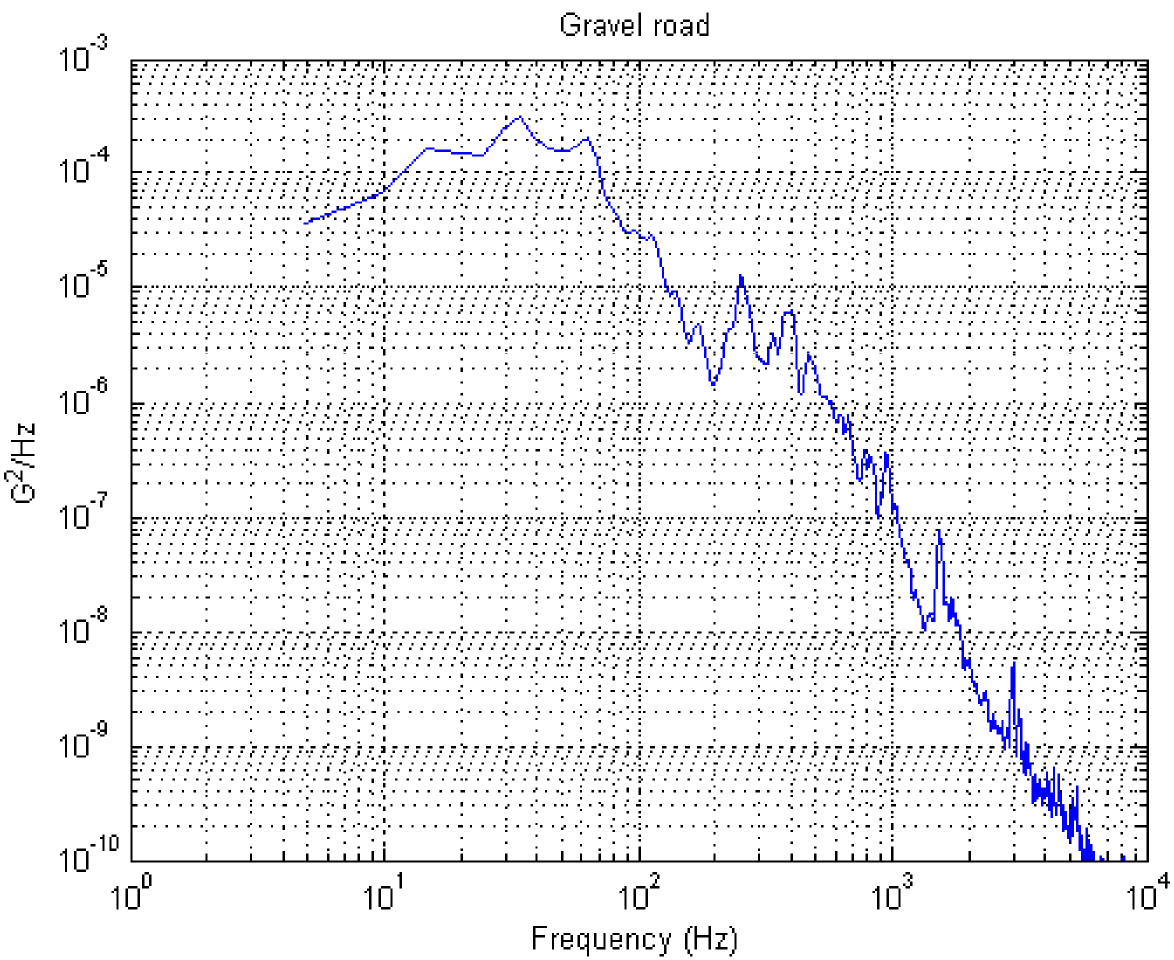
60 seconds of K=5.7 "Kurtosion" random



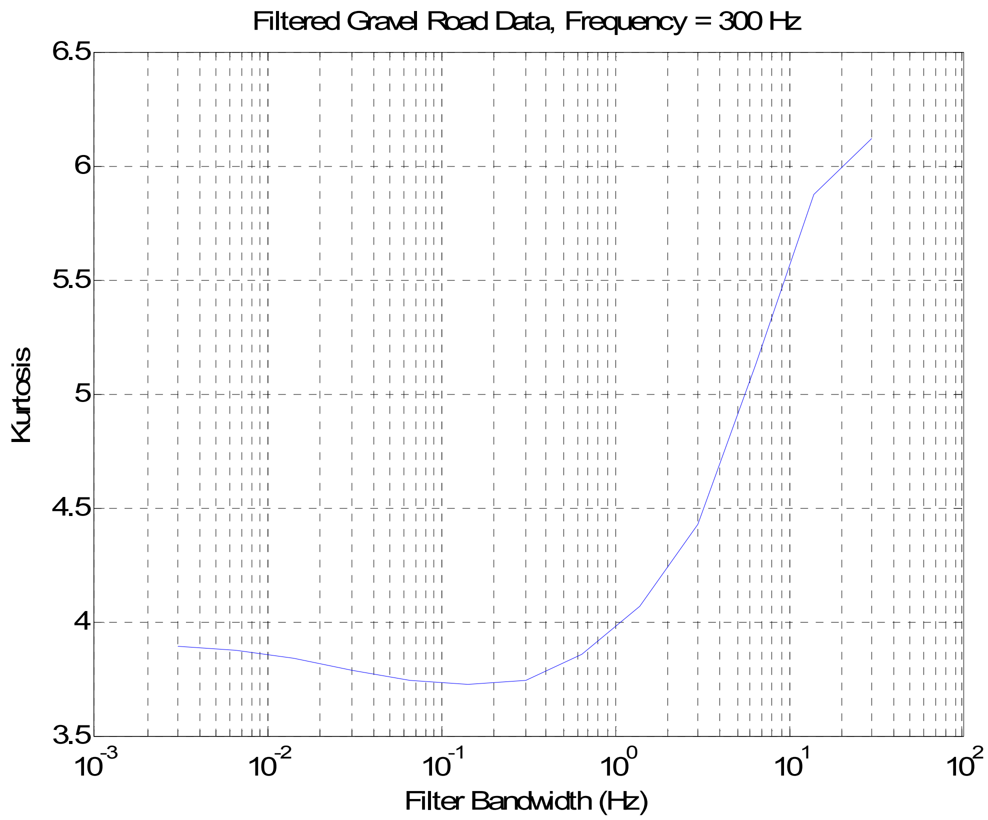
Gravel Road Vibration Waveform



Gravel Road Spectrum



Effect of Resonance on Kurtosis



Conclusions

- Papoulis rule, while true, only forces pure Gaussian random on an infinitely narrow resonance.
- You can increase the kurtosis of the vibration even at a product's resonance by paying attention to the transition frequency.
- To significantly increase the reliability of your random test, you should correlate your kurtosis to real-world measured events.
- To significantly accelerate the failure of your product during testing, you should increase the kurtosis past the standard of k = 3 that is used today.

References

- Van Baren, John "How to get Random Peaks Back to 120G," *TEST Engineering & Management,* August/September 2007.
- Van Baren, John "Kurtosis: The Missing Dashboard Knob," *TEST Engineering & Management,* October/November 2005, pp 14-16.
- Van Baren, Philip "The Missing Knob on Your Random Vibration Controller," *Sound and Vibration, October 2005,* pp 10-17.
- Smallwood, David O.: "Generating Non-Gaussian Vibration for Testing Purposes," *Sound and Vibration, October 2005,* pp 18-23.
- Connon, W.H., "Comments on Kurtosis of Military Vehicle Vibration Data," *Journal of the Institute of Environmental Sciences*, September/October 1991, pp 38-41.
- Steinwolf, A. , "Shaker simulation of random vibration with a high kurtosis value," *Journal of the Institute of Environmental Sciences*, May/Jun 1997, pp 33-43.
- Steinwolf, A. and Connon W.H., "Limitations on the use of Fourier transform approach to describe test course profiles," *Sound and Vibration*, Feb. 2005, pp. 12-17.
- www.sandv.com has PDFs of *Sound and Vibration* articles.

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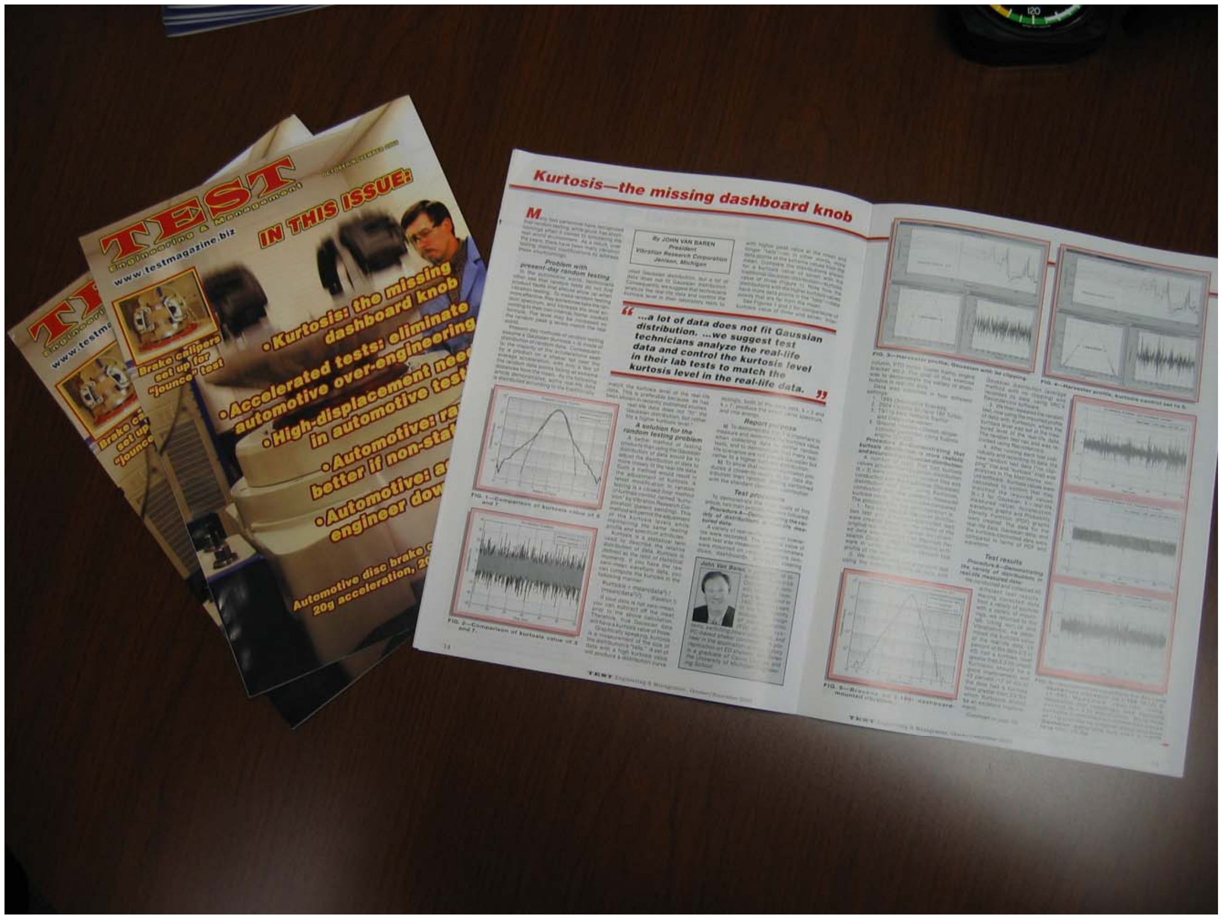

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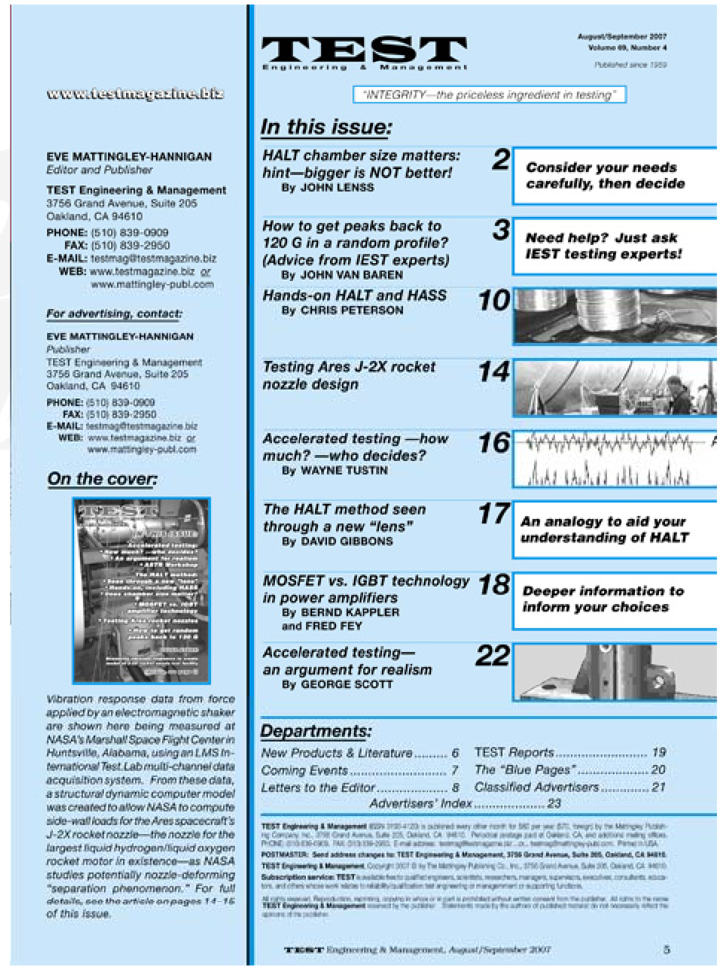

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On the cover:

Vibration response data from force applied by an electromagnetic shaker are shown here being measured at NASA's Marshall Space Flight Center in Huntsville, Alabama, using an LMS International Test.Lab multi-channel data acquisition system. From these data, a structural dynamic computer model was created to allow NASA to compute side-wall loads for the Ares spacecraft's J-2X rocket nozzle—the nozzle for the largest liquid hydrogen/liquid oxygen rocket motor in existence—as NASA studies potentially nozzle-deforming "separation phenomenon." For full details, see the article on pages 14–16 of this issue.

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