Random Vibration Kurtosis Control

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What is kurtosis?
Why are we interested in kurtosis?
Kurtosis in the resonance?
Papoulis Rule / Central Limit Theorem.
Test-Shaker with Resonating bar.
Control Random ED shaker kurtosis?
How does this relate to the real world?
The Problem

Definition of the Problem

- Traditional random testing does not always find failures that occur during the life of a product.
- This is likely because the product experiences high G forces in actual use that are higher than traditional random testing generates.
Kurtosis

♦ Show of hands: Who has heard of the term “Kurtosis” before today?

♦ Definition in terms of statistical moments
  ➢ Mean is the 1\textsuperscript{st} moment
  ➢ Variance is the 2\textsuperscript{nd} moment
  ➢ Skewness is normalized 3\textsuperscript{rd} central moment
  ➢ Kurtosis is normalized 4\textsuperscript{th} central moment
Calculating Kurtosis

The basic function for calculating kurtosis for zero-mean data is:
\[
\frac{\text{average}(data^4)}{\left(\text{average}(data^2)^2\right)^2}
\]

Different people normalize this value in different ways

- As commonly used, Gaussian kurtosis = 3
- Microsoft Excel subtracts 3, so Gaussian kurtosis = 0
- Others divide by 3, so Gaussian kurtosis = 1
Traditional Random Testing

♦ Current random testing seeks to achieve a Gaussian distribution
  - “Normal” distribution
  - Concentrated around mean
  - Low probability of extreme values
  - Kurtosis = 3
What is Missing on ED Shaker Controllers?

♦ Random vibration controllers have 2 basic “knobs”:
  ➢ Frequency content - Power Spectral Density (PSD)
  ➢ Amplitude level - RMS

♦ Need a third ‘knob’ to adjust the Kurtosis
  ➢ Allows adjustment of the PDF (probability density function)
  ➢ Increasing kurtosis = increasing peak levels
  ➢ Allows the damage-producing potential of the test to be adjusted independent of the other two controls.
The Missing Knob
Kurtosis Control

♦ Objective is to control the amplitude distribution to achieve the higher peaks seen in field data
  - Spectrum is a measure of the frequency content
  - RMS is a measure of the amplitude
  - Kurtosis is a measure of the “peakiness”

♦ Solution is use a non-Gaussian vibration and control the Kurtosis

♦ This is what we call Kurtosion™
  - Method to simultaneously control Spectrum, RMS, and Kurtosis
  - Patent-pending
Increased Kurtosis = More Time at Peaks

Kurtosis = 3 is > 3σ
0.27% of time

Kurtosis = 4 is > 3σ
0.83% of time

Kurtosis = 7 is > 3σ
1.5% of time

Note: 1.5% of a 1 hour test is nearly a full minute above 3σ
Increased Kurtosis = Higher Peaks

Crest Factor:
The ratio of peak to rms

Note:
Crest factor of the 99.994% probability level is plotted, as this is the 4 times rms (4 sigma) level for a kurtosis=3 random test. Also, this is the typical maximum peak seen on a kurtosis=3 random test.
Waveform Comparison
Useful Properties for Kurtosis Control

- Set kurtosis independent of RMS.
- Set kurtosis without affecting PSD.
- Increase kurtosis over the full spectrum.
- Dynamic range of the controller is maintained.
- Apply kurtosis even in a resonance.
- Note Papoulis Rule requirements.
Papoulis’ Rule states that filtered waveform *tends towards* Gaussian

- Bound is proportional to the 14th root of the filter bandwidth. This is an extremely weak limit.
- Bound is constant across all values of the CDF. Weak limit on the tails of the distribution which give Kurtosis

Practical results of Papoulis’ Rule

- Kurtosis at the resonance gets reduced from the kurtosis of the excitation signal
- For practical Q factors, there will still be some significant kurtosis at the resonance.
Resonating Bar Test Setup
10,000 Hz Transition Frequency

Acceleration Profile

Probability Density Function

Kurtosis vs. Time
1,000 Hz Transition Frequency

Acceleration Profile

Probability Density Function

Kurtosis vs. Time

Kurtosis vs. Time

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100 Hz transition Frequency

Acceleration Profile

Probability Density Function

Kurtosis vs. Time

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10 Hz Transition Frequency

Acceleration Profile

Probability Density Function

Kurtosis vs. Time

K ch1: 6.38
K ch2: 5.98

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Effect of Transition Frequency on PDF

PDF for the Brass Bar (Arm) Data for tests with Gaussian distribution, Kurtosis = 5 (Transition 10,000 Hz) and Kurtosis = 5 (Transition 10 Hz).
Effect of Transition Frequency on Kurtosis

Kurtosis vs. Transition Frequency on Brass Bar

- Shaker head
- Brass bar

Kurtosis vs. Transition Frequency (Hz)

Kurtosis

3

3.5

4

4.5

5

5.5

10^1

10^2

10^3

10^4
Control on Resonant Beam

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>TRANSITION FREQUENCY</th>
<th>KURTOSIS SETTING</th>
<th>KURTOSIS HEAD</th>
<th>KURTOSIS ARM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brass Bar</td>
<td>10-2000 Hz</td>
<td>10000 Hz</td>
<td>5</td>
<td>10.5</td>
</tr>
<tr>
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<td>10-2000 Hz</td>
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<tr>
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<td>5</td>
<td>5.78</td>
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</tbody>
</table>

Results from Brass Bar Vibrations where the end of the bar was controlled and the head responded.

Lower Transition Frequency allows controller to easily produce the desired kurtosis value at resonance.
Light Bulb Test - revisited

- Previous papers examined the effect of increasing kurtosis on failure of light bulbs.
- As kurtosis was increased, failure time decreased.
- Now, we run a test with constant kurtosis, and vary the transition frequency.
Consider the Kurtosis Comparison Chart shown here for the 22 bulbs at each kurtosis level.

The time it took to complete a test decreased dramatically as the kurtosis level increased.
Consider the graph of the Mean Minutes to Failure vs. Kurtosis Level as shown here.

The increased kurtosis level dramatically decreased the mean time it took for the light bulbs to fail.
New Light Bulb Test
Minutes to Failure

<table>
<thead>
<tr>
<th>Gaussian 1.00 X RMS</th>
<th>Kurtosis 5 TF: 10000 Hz</th>
<th>Kurtosis 5 TF: 100 Hz</th>
<th>Kurtosis 5 TF: 10 Hz</th>
<th>Kurtosis 5 TF = 1 Hz</th>
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<tbody>
<tr>
<td>32</td>
<td>7</td>
<td>12</td>
<td>6</td>
<td>2</td>
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<td>62</td>
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<td>4</td>
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<td>10</td>
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<tr>
<td><strong>50.9</strong></td>
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<td><strong>14.9</strong></td>
<td><strong>12.3</strong></td>
<td><strong>5.3</strong></td>
</tr>
</tbody>
</table>

Light bulb failure times at different Transition Frequency values. Note that as the Transition Frequency value decreases the time to failure also decreases.
### Minutes to Failure

#### Average Time for Light bulb Failure

- **Gaussian:**
  - $K=5$
  - $TF=10$ kHz
- $K=5$
- $TF=100$ Hz
- $K=5$
- $TF=10$ Hz
- $K=5$
- $TF=1$ Hz

*Varied Transition Frequency*
What is the Kurtosis of the Real World?

Probability Density Function

- Gaussian
- Gravel road
- Highway
- Paved road
- Rumble strip
- Airplane

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PDF of a Real Life Environment

Oldsmobile Bravada, dashboard vibration
As vehicle travels down I-196.

Probability Density Function for Gaussian, and actual, and controlled kurtosis
Random Test with PDF of a Real Life Environment

Spectrum defined by field measured data
Traditional random test with 3 sigma clipping

Same spectrum defined by field measured data
Now with Kurtosis Control set to 5
60 seconds of some actual field data $K=5.7$

60 seconds of some Traditional random $K=3$

60 seconds of $K=5.7$ “Kurtosion” random
Effect of Resonance on Kurtosis

Filtered Gravel Road Data, Frequency = 300 Hz
Conclusions

♦ Papoulis rule, while true, only forces pure Gaussian random on an infinitely narrow resonance.
♦ You can increase the kurtosis of the vibration even at a product’s resonance by paying attention to the transition frequency.
♦ To significantly increase the reliability of your random test, you should correlate your kurtosis to real-world measured events.
♦ To significantly accelerate the failure of your product during testing, you should increase the kurtosis past the standard of $k = 3$ that is used today.
References

♦ [www.sandv.com](http://www.sandv.com) has PDFs of *Sound and Vibration* articles.
IN THIS ISSUE:

- ASTR Workshop
- The HALT method: Seen through a new "lens"
- Hands-on, including HASS
- Does chamber size matter?
- MOSFET vs. IGBT amplifier technology
- Testing Ares rocket nozzles
- How to get random peaks back to 120 G

COVER STORY:
Measuring vibration responses to create model of J-2X rocket nozzle test facility

In this issue:

- HALT chamber size matters: bigness is NOT better? by JOHN LINDSAY
- How to get HALT back to 120G in a random profile? (Advice from TEST experts) by JOHN VAN EMMEERHANDS-on HALT AND HASS by CHRIS PETERSON
- HALT testing Ares rocket nozzle design by WAYNE TEUTTINI
- Accelerated testing — how much? — who decides? by WAYNE TEUTTINI
- The HALT method seen through a new "lens" by DAVID GIBBONS
- MOSFET vs. IGBT technology in power amplifiers by EMIL KAPLAR and FRED TEY
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Thank You

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