A Primer on Fatigue Damage and Fatigue Damage Spectra

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Background/Theory

Fatigue is the result of multiple repetitions of low-level stress applied to an object. Generally, fatigue happens in three stages: crack initiation; crack propagation; and final fracture. These three stages can clearly be seen in examples of aluminum beams that experienced various levels of fatigue during experimental testing (Figures 1-3).

![Figure 1: Crack initiation](image1)
![Figure 2: Crack propagation](image2)
![Figure 3: Beam fracture due to fatigue](image3)

All structures experience fatigue as they are repeatedly exposed to adequate levels of stress. This fatigue accumulates over time as the product continues to experience these stress levels. This accumulation of fatigue is known as Miner’s Rule of Damage. The total damage a product experiences in a particular time period can be calculated from field data PSD (energy of vibration test) and plotted for a specific range of frequencies. The resulting plot of fatigue damage versus frequency is the Fatigue Damage Spectrum (FDS) – a means by which to quantify the stress-strain loads placed on a product.

Calculating Fatigue Damage and the Fatigue Damage Spectrum

The Fatigue Damage Spectrum is produced by plotting the individually calculated fatigue damage values for narrow frequency bands. For example, the PSD data from a particular test is run through a narrowband filter, utilizing a specific Q value. A specialized calculation tool is then used to determine the fatigue damage for the data filtered for each frequency band. This is accomplished by using a Rainflow counting algorithm to count the stress peak-valley cycles. The stress cycle amplitudes are weighted non-linearly, because of the power law function found in Miner’s rule \(N = cS^{-p}\). “The most commonly used method for calculating a reduction in test duration is the Miner-Palmgren hypothesis that uses a fatigue-based power law relationship to relate exposure time and amplitude” (MIL-STD-810 G; Method 514.6, Annex A). These cycles are accumulated to get the accumulated fatigue at that specific frequency, according to Henderson-Piersol’s fatigue calculation method. At this point, since the “Q” of the resonance has been specified, as well as the “b” value (assumed to be the slope of the S-N curve for the material composing the UUT), the fatigue damage value for each frequency can be calculated. The collective plot of all of these fatigue damage values is the Fatigue Damage Spectrum (Figure 4).
Fatigue Damage Analogies

a. River Analogy

As river water flows it slowly erodes away the banks of the river. Perhaps the water itself rubs across the soil and knocks particles loose. Or, perhaps the water carries other material (tree branches or sediment) that erodes away the riverbank. One way or another, the riverbank erodes. But, if the river has a faster current, it will erode away the riverbank more quickly. River water with greater velocity has more energy to dislodge rocks and pebbles from the riverbank soil. Consequently, a faster flowing river will erode the riverbanks more quickly. If one knows the rate at which the soil erodes under normal river flow, one should be able to calculate the rate at which the soil will erode with a regularly faster flowing river.

This natural phenomenon illustrates nicely the similar phenomenon of fatigue damage. Suppose we consider breaking a paperclip by bending it back and forth multiple times. There are numerous bending modes for the paperclip: bend it back and forth with a maximum angle of 10°; of 20°; of 30° and so on. Perhaps 100 bends at 10°, plus 25 bends at 30° will cause the paperclip to break. That means that the paperclip has accumulated its life-dose of fatigue. That same life-dose of fatigue also could be accomplished by perhaps 30 bends at 20° plus 40 bends at 40°. Either way, according to Miner’s Rule of Damage, the paperclip has accumulated its life-dose of fatigue. This is analogous to the river water flowing at a regular rate, with some combination of either the water or its sediments doing the eroding.

How can one break the paperclip more quickly? How can the bending paperclip be related to the increased flow of river water? To understand this we must change the focus of our analogy from bending the paperclip through various angles to the vibrating of the paperclip at various frequencies.
Suppose we find the main resonant frequency of the paperclip. If we vibrate at that resonant frequency, the life-dose of fatigue for the paperclip will be reached after a specific number of vibration cycles. To bring the paperclip to its life-dose of fatigue more quickly, we must vibrate the paperclip at its resonant frequency with stronger accelerations (akin to water flowing with more speed (energy)). In the end, the paperclip breaks when it experiences its life-dose of fatigue. That life-dose of fatigue can be accumulated gradually (low level accelerations at the resonant frequency) or rapidly (high level accelerations at the resonant frequency). In the end, however, the paper clip breaks when it experiences its life-dose of fatigue; regardless of how it arrived at that life-dose level.

b. Water Bucket Analogy

To explain Fatigue Damage and the Fatigue Damage Spectrum in a different way, consider a “water bucket” analogy. The Fatigue Damage Spectrum is a plot of the amount of fatigue at every frequency in the spectrum that will bring the product to failure (Figure 5).

![Figure 5: A typical FDS plot.](image)

That is, the total life-dose of fatigue for the object under test is the area under the FDS curve. To help visualize this idea from a different perspective, imagine if we flipped the plot upside-down (Figure 6).
The inverted FDS plot is akin to a “bucket”. If we fill the bucket with water (fatigue) then the object under test will be brought to failure when the bucket becomes full (life-dose of fatigue). The total life-dose of fatigue is represented by the amount of water necessary to fill the bucket (Figure 7).

Now to accelerate the test (i.e. to bring the product to failure more quickly), we simply need to fill the bucket with water more quickly. The FDS test accomplishes this by applying a higher level of GRMS to the test. The goal is not to add more water to the bucket (increasing the amount of total fatigue), but to change the rate at which the water is added to the bucket (increase the rate at which fatigue accumulates).
Using Fatigue Damage Spectrum Software

a. Importing and Accelerating Tests

Using the Random Import Tab (choosing Fatigue Damage Import Method), the test engineer imports a recorded file and sets various fatigue parameters (m value and Q value) and enters the expected Target Life and the desired Test Duration. The software computes a FDS for that recorded file and its expected Target Life (Figure 8).

![Random Test Settings](image)

**Figure 8:** Random Import Tab for Fatigue Damage Spectrum
To accelerate the test, the test engineer must choose a shorter Test Duration. To bring the product to failure for this newly desired Test Duration, the GRMS value must increase, resulting in an increase in the PSD (Figure 9).

Figure 9: Random Import Tab for FDS, showing the increased PSD and GRMS values for a decreased Test Duration
With this newly desired Test Duration, the FDS spectrum does not change. As long as the Target Life value remains the same, the total Fatigue damage will remain the same. Decreasing the Test Duration, however, will result in accumulating the total fatigue damage \textit{in a quicker time}. This is accomplished by an increase in GRMS (Figure 10).

\textbf{Figure 10:} FDS spectra are identical for imported recordings with the same Target Life. Changing the Test Duration will result in a change in GRMS but not the life-dose of fatigue of the product. With the increased GRMS level the life-dose of fatigue will accumulate more quickly.

\textbf{b. Combining Different Environments}

The real-world environment for products is not a simple vibration profile. Products typically experience a wide-range of environments. With Fatigue Damage Import, test engineers can combine various vibration environments to more accurately simulate the end-use environment for a product.

Suppose you live in Hudsonville, MI. You likely take a number of roads each day as you travel to work or school or to the store. We collected data on a variety of roads in Hudsonville and imported them into a random test with the Fatigue Damage Import (Figure 11).
Figure 11: 4 different road environments were imported one at a time into the Fatigue Damage Import with m=4.8 and Q=50.

To create a profile that combines all those environments and is able to simulate the real-life vibrations experienced by our vehicle in its daily trips in Hudsonville, we adjusted the Target Life to represent the relative amount of time the vehicle is on the various roads (Figure 12). The resulting “combined” profile changed shape slightly as it was influenced more heavily by the large number of passes on some roads than others.

Figure 12: Different road environments are experienced for different amounts of time. Consequently, the Target Life value was adjusted for each road to compensate for these differences. Note how the “combined” profile is different than the “combined” profile in Figure 11 (especially between 50 and 100 Hz; in terms of shape and magnitude).
To generate this “combined” profile, a test engineer should click the “Create Table” button. To accelerate the test of this profile, the test engineer should enter a reasonable Test Duration value (Figure 13). This new test duration value will cause the PSD plot and the RMS value to increase. In order to accumulate the life-dose of fatigue in a shorter amount of time, the test must necessarily run at a higher GRMS value.

![Random Test Settings](Image)

**Figure 13:** The four road test profiles are combined into one representative profile (by clicking Create Table – green line in profile). The test is accelerated by adjusting the Test Duration value. Note how a new increased PSD profile is formed (blue line in profile).

In addition, the test engineer can utilize Kurtosion® to further accelerate the test and make it more realistic. When the files are imported with the Fatigue Damage Import, the PSD that is finally generated from the Fatigue Damage Spectrum is Gaussian in nature due to the calculation method employed by the Henderson-Piersol fatigue damage calculation². Since real-world vibrations are not always Gaussian in nature, the test engineer may desire to adjust the Kurtosis value to match the real-world environment. Since a higher kurtosis value will reintroduce into the test the large peaks that are removed in a Gaussian calculation, the test will include more of the large peak accelerations that are the chief cause of damage to a product. Consequently, by increasing the Kurtosis value the test will achieve the life-dose of fatigue **more quickly**. It will do so, however, with a **lower GRMS value** (Figure 14).
Summary

The fatigue (stress) experienced by a unit under test (UUT) accumulates according to Miner’s Rule. VRC’s Fatigue Damage Spectrum Random Import option uses this concept to bring a product to failure more quickly by decreasing its Test Duration. This decrease in Test Duration, increases the GRMS levels, accumulating fatigue at a faster rate. The UUT receives its life-dose of fatigue at a faster rate when the Test Duration level decreases.

In addition test engineers can use the Fatigue Damage Import to combine many different vibration environments into one test profile. The test engineer can also accelerate the test made from the new combined profile by adjusting the Test Duration value. Finally, the test engineer can apply Kurtosis Control (Kurtosion®) to the test profile, further accelerating the test and making it more realistic.

Overall, the Fatigue Damage Import is a very valuable tool. It can be used by test engineers to accurately simulate the end-use environment for a UUT; to accelerate a test to a desired test duration; and to combine many different real-life environments into one unique FDS profile. The Fatigue Damage Import can also be used in conjunction with Kurtosion® in order to further accelerate a test and make the test more realistic by including the kind of peak accelerations that are really found in the end-use environment.

Figure 14: The combined profile of the four road tests is further adjusted by utilizing Kurtosion®. Note that by increasing the kurtosis value, the combined RMS value decreased (compare GRMS values from Figure 13 and 14).
References
